SRI MANAKULA VINAYAGAR ENGINEERING COLLEGE
(Approved by AICTE, New Delhi & Affiliated to Pondicherry University)
(Accredited by NBA-AICTE, New Delhi, ISO 9001:2000 Certified Institution & Accredited by NAAC with "A" Grade)
Madagadipet, Puducherry - 605 107

DEPARTMENT OF MECHANICAL ENGINEERING

Lecture Notes

MATHEMATICS-IV

II Year/ IV Semester

PREPARED BY

N. VIJAYAN

Department of Mathematics

SMVEC

CONTENTS

1.Syllabus

2.Part B Questions and Answers

3.Part-A Questions and Answers

4.Tutorial Questions

5.University Question Papers

6.Solved University Question Papers

7. Statistical Tables

8. List of formulae

MAT41 MATHEMATICS – IV (3 1 0 4)

Partial Differential Equations

Unit – I

Formation of PDE by elimination of arbitrary constants and arbitrary functions – General singular. Particular and complete integrals – Lagrange's linear first order equation – Higher order differential equations with constant coefficients. (12 hours)

Unit - II

Solution of partial differential equation by the method of separation of variables – Boundary value problems – Fourier series solutions – Transverse vibration of an elastic string. (12 hours)

Unit – III

Fourier series solution for one dimensional heat flow equation – Fourier series solutions for two dimensional heat flow equations under steady state conditions (Cartesian and polar forms). (12 hours)

Applied Statistics

Unit – IV

Curve fitting by the method of least squares – fitting of straight lines, second degree parabolas and more general curves. Test of significance: Large sample test for single proportion, difference of proportions, single mean, difference of means, and difference of standard deviations. (12 hours)

Unit – V

Small samples: Test for single mean, difference of means and correlation coefficients – test for ratio of variances – Chi–Square test for goodness of fit and independence of attributes. (12 hours)

TEXT BOOK:

1. M.K.Venkataraman, Engineering Mathematics, Vol. II & III, National Publishing Co., Madras, 2007.

2. S.C. Gupta & V.K. Kapoor : Fundamentals of Mathematical Statistics, Sultan Chand Sons , New-Delhi,
2008.

REFERENCE BOOKS:

1. N.P. Bali & Manish Goyal : A text book of Engineering Mathematics, Laxmi Publications, New- Delhi,
2008.

2. Erwin Kreyszig : Advanced Engineering Mathematics, John- Wiley sons, New-York, 2005.

3. B. S. Grewal, : Higher Engineering Mathematics, Khanna Publishers, New-Delhi, 2008.

UNIT I

PARTIAL DIFFERENTIAL EQUATION (PDE)

TYPE 1:

Elimination of Arbitrary constants

1. **Form the PDE by eliminating the arbitrary constants $z=ax+by+a^2+b^2$**

Given: $z = ax + by + a^2 + b^2$----------(1)

Diff w.r.to 'x'

$$\frac{\partial z}{\partial x} = a + 0$$

p =a'

Diff w.r.to 'y'

$$\frac{\partial z}{\partial y} = b + 0$$

q =b

Sub a and b value in (1)

$Z=px+qy+p^2+q^2$

2.Form the PDE by eliminating the constants $z=(x-a)^2+(y-b)^2+1$

Given: $z = (x - a)^2 + (y - b)^2 + 1$ ----------(1)

Diff w.r.to 'x'

$$\frac{\partial z}{\partial x} = 2(x-a)$$

$$(x-a) = \frac{p}{2} \qquad \text{---------- (2)}$$

Diff w.r.to 'y'

$$\frac{\partial z}{\partial y} = 2(y-b)$$

$$(y-b) = \frac{q}{2}$$

Sub in (1)

$$z=\left(\frac{p}{2}\right)^2+\left(\frac{q}{2}\right)^2+1$$

$$z=\frac{p^2}{4}+\frac{q^2}{4}+1$$

$4z=p^2+q^2+4$

3. **Form the PDE $z=(x^2+a)(y^2+b)$**

Given: PDE $z = (x^2 + a) (y^2 + b)$----------(1)

Diff w.r.to 'x'

$$\frac{\partial z}{\partial x}=2x\left(y^2+b\right)$$

$$\left(y^2+b\right)=\frac{p}{2x}\text{--------}\rightarrow(2)$$

Diff w.r.to 'y'

$$\frac{\partial z}{\partial y}=2y\left(x^2+a\right)$$

$$\left(x^2+a\right)=\frac{q}{2y}\text{---------}\rightarrow(3)$$

Substitute eqn (2) and (3) in (1)

$$z=\frac{p}{2x}\times\frac{q}{2y}$$

$$z=\frac{pq}{4xy}$$

$$4xyz=pq$$

4. **Form the PDE of $\log(az-1)=x+ay+b$**

Given: $\log(az-1) = x + ay + b$ ----------1

Diff w.r.to 'x'

$$\frac{1}{az-1}a\frac{\partial z}{\partial x}=1$$

$$\frac{1}{az-1}ap=1$$

$$ap=az-1\text{----------}2$$

Diff w.r.to 'y'

$$\frac{1}{az-1}a\frac{\partial z}{\partial y}=a$$

$$\frac{1}{az-1}q=1$$

$$q=az-1\text{----------}3$$

Equating 2 and 3

$$ap=q$$

$$a=\frac{q}{p}$$

Sub in 3

$$q=\frac{q}{p}z-1$$

$$pq=qz-p$$

$$qz=pq+p$$

$$qz=p(q+1)$$

5. Form the PDE $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Given: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$**----------1**

Diff pw.r.to 'x'

$$\frac{2x}{a^2}+\frac{2z}{c^2}\frac{\partial z}{\partial x}=0$$

$$\frac{2z}{c^2}p=-\frac{2x}{a^2}$$

$$\frac{a^2}{c^2} = -\frac{x}{pz}$$

$$\frac{c^2}{a^2} = -\frac{pz}{x}$$ **----------2**

Diffp w.r.to 'y'

$$\frac{2y}{b^2} + \frac{2z}{c^2}\frac{\partial z}{\partial y} = 0$$

$$\frac{2z}{c^2}q = -\frac{2y}{b^2}$$

$$\frac{b^2}{c^2} = -\frac{y}{qz}$$

$$\frac{c^2}{b^2} = -\frac{qz}{y}$$ **----------3**

Diff p w.r.t ''x'' in eqn 2

$$\frac{2}{a^2} + \frac{2}{c^2}\left[z\frac{\partial^2 z}{\partial x^2} + p^2\right] = 0$$

$$\frac{1}{c^2}\left[z\frac{\partial^2 z}{\partial x^2} + p^2\right] = -\frac{1}{a^2}$$

$$z\frac{\partial^2 z}{\partial x^2} + p^2 = -\frac{c^2}{a^2}$$

Substitute (2)in above equation

$$z\frac{\partial^2 z}{\partial x^2} + p^2 = \frac{zp}{x}$$

$$xzr + xp^2 = zp$$

Which is the required solutions.

6. Form the PDE $z^2\cot^2\alpha = (x-a)^2 + (y-b)^2$

Given: $z^2\cot^2\alpha = (x-a)^2 + (y-b)^2$ ----------1

Diff pw.r.t.'x'

$$2z\frac{\partial z}{\partial x}\cot^2\alpha = 2(x-a)$$

$$zp\cot^2\alpha = (x-a) \text{ -----} \rightarrow (2)$$

diff.p. w.r.to ý',

$$2z\frac{\partial z}{\partial y}\cot^2\alpha = 2(y-b)$$

$$zq\cot^2\alpha = (y-b) \text{ -----} \rightarrow (3)$$

Sub 2 and 3 in 1

$$z^2\cot^2\alpha = \left(zp\cot^2\alpha\right)^2 + \left(zq\cot^2\alpha\right)^2$$

$$z^2\cot^2\alpha = z^2p^2\cot^4\alpha + z^2q^2\cot^4\alpha$$

$$1 = \left(p^2+q^2\right)\cot^2\alpha$$

$$p^2+q^2 = \tan^2\alpha$$

Which is the required solutions.

7. **Form the PDE of sphere whose centre lies on zaxis**
   Solution:

The sphere passing through z axis eqn is written as

$$x^2+y^2+(z-c)^2 = r^2 \text{ -----} \rightarrow (1)$$

` D.p.w.r t "x"

$$2x+2(z-c)p = 0$$

$$x+(z-c)p = 0$$

$$z-c = -\frac{x}{p} \text{ -------} (2)$$

D. p. w.r t "y"

$$2y+2(z-c)q = 0$$

$$y+(z-c)q=0$$

$$z-c=-\frac{y}{q}------(3)$$

Equating 2 and 3

$$-\frac{x}{p}=-\frac{y}{q}$$

$$xq-yp=0$$

Which is the required solutions.

TYPE-II

Elimination of Arbitrary Function:

Method 1:

If we eliminate only 1 arbitrary function from the given equation, we get first order PDE.

1. Form the PDE by eliminating Arbitrary function, $z = f(x^2 + y^2 + z^2)$

<u>Given</u>:

$$Z = f(x^2 + y^2 + z^2) \qquad \ldots\ldots..1$$

D. p. w. r. to x,

$$p = f'(x^2+y^2+z^2)(2x+2zp)$$

$$\frac{p}{2x+2zp} = f'(x^2+y^2+z^2) ------\rightarrow (2)$$

D.p.w.r.to y,

$$q = f'(x^2+y^2+z^2)(2y+2zq)$$

$$\frac{q}{2y+2zq} = f'(x^2+y^2+z^2) ------\rightarrow (3)$$

Equating 2 and 3,

$$\frac{q}{2y+2zq} = \frac{p}{2x+2zp}$$

p(2y+2zq)= q(2x+2zp)

2py+2zpq-2xq-2zpq=0

py – qx=0

Which is the required solutions.

2. **Form the PDE by eliminating arbitrary function** $z=(x+y)\phi(x^2-y^2)$

<u>Given:</u>

$$z=(x+y)\phi(x^2-y^2)\ldots\ldots\ldots\ldots 1$$

D.p.w.r.to x,

$$p=(x+y)\phi'(x^2-y^2)2x+\phi(x^2-y^2).1 \qquad \ldots\ldots\ldots\ldots 2$$

D.p.w.r.to y,

$$q=(x+y)\phi'(x^2-y^2)2y+\phi(x^2-y^2)(1) \quad \ldots\ldots\ldots\ldots 3$$

Eqn. (2)y and eqn. (3)x,

$$yp=2xy(x+y)\phi'(x^2-y^2)+y\phi(x^2-y^2)$$

$$qx=2xy(x+y)\phi'(x^2-y^2)+x\phi(x^2-y^2)$$

$$----------$$

$$py+qx=y\phi(x^2-y^2)+x\phi(x^2-y^2)$$

$$py+qx=\phi(x^2-y^2)(x+y) \qquad \ldots\ldots\ldots\ldots 4$$

Substitute (4) in (1)

$$py+qx=z$$

Which is the required solutions

3. Form the PDE by eliminating arbitrary function, $z=f\left(\frac{xy}{z}\right)$

<u>Given</u>:

$$z=f\left(\frac{xy}{z}\right)\ldots\ldots\ldots\ldots\ldots 1$$

D.p.w.r.to x,

$$p = f'\left(\frac{xy}{z}\right)\left(\frac{zy - xyp}{z^2}\right)$$

$$f'\left(\frac{xy}{z}\right) = \left(\frac{pz^2}{zy - xyp}\right) \qquad \ldots\ldots\ldots\ldots\ldots 2$$

D.p.w.r.to y,

$$q = f'\left(\frac{xy}{z}\right)\left(\frac{zx - xyq}{z^2}\right)$$

$$f'\left(\frac{xy}{z}\right) = \left(\frac{qz^2}{zx - xyq}\right) \ldots\ldots\ldots\ldots 3$$

Equating 2 and 3,

$$\left(\frac{pz^2}{zy - xyp}\right) = \left(\frac{qz^2}{zx - xyq}\right)$$

$$pz^2\left(zx - xyq\right) = qz^2\left(zy - xyp\right)$$

$$pzx - xypq = qzy - xypq$$

$$px = qy$$

$$px - qy = 0$$

Which is the required solutions

4. Form the PDE by eliminating arbitrary function $xyz = \phi(x + y + z)$

Solution:

<u>Given:</u>

$$xyz = \phi(x + y + z) \qquad \ldots\ldots\ldots\ldots 1$$

D.p.w.r.to x,

$$y\left(x\frac{\partial z}{\partial x} + z\right) = \phi'\left(x + y + z\right)\left(1 + \frac{\partial z}{\partial x}\right)$$

$$y\left(xp + z\right) = \phi'\left(x + y + z\right)\left(1 + p\right)$$

$$\phi'\left(x + y + z\right) = \frac{y\left(xp + z\right)}{\left(1 + p\right)} \qquad \ldots\ldots\ldots 2$$

D.p.w.r.to y,

$$x\left(y\frac{\partial z}{\partial y}+z\right)=\phi'(x+y+z)\left(1+\frac{\partial z}{\partial y}\right)$$

$$x(yq+z)=\phi'(x+y+z)(1+q)$$

$$\phi'(x+y+z)=\frac{x(yq+z)}{(1+q)} \qquad \text{……..3}$$

Equating 2 and 3,

$$\frac{y(xp+z)}{(1+p)}=\frac{x(yq+z)}{(1+q)}$$

$$(xyp+zy)(1+q)=(xyq+zx)(1+p)$$

$$xypq+xyp+zy+zyq=xypq+xyq+zx+zxp$$

$$xyp+zy+zyq-xyq-zx-zxp=0$$

$$xp(y-z)+yq(z-x)=z(x-y)$$

Which is the required solutions

METHOD 2:

Formation of PDE by eliminating of arbitrary function $\emptyset$ drom $\emptyset(\boldsymbol{u},\boldsymbol{v})=\mathbf{0}$

Let (u, v) be any two functions of x, y and z (i. e, $\emptyset(u,v)=0$)

To eliminate the arbitrary function, we use the formula, $\begin{vmatrix} u_x & v_x \\ u_y & v_y \end{vmatrix}=0$

1. **Form the PDE by eliminating arbitrary function,**

Solution: $\phi(x^2+y^2+z^2, x+y+z)=0$

The given eq. is of the form $\emptyset(u,v)=0$

Where,

u =$x^2+y^2+z^2$

v = x+y+z

$$u_x = 2x + 2z\frac{\partial z}{\partial x} \qquad v_x = 1 + \frac{\partial z}{\partial x}$$

$$u_x = 2(x + zp) \qquad v_x = 1 + p$$

$$u_y = 2y + 2z\frac{\partial z}{\partial y} \qquad v_y = 1 + \frac{\partial z}{\partial y}$$

$$u_y = 2(y + zq) \qquad v_y = 1 + q$$

w.k.t,

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

$$\begin{vmatrix} 2x + 2zp & 1 + p \\ 2y + 2zq & 1 + q \end{vmatrix} = 0$$

(1+q) (2x+2zp) – (1+p) (2y+2zq)=0

2x + 2xq + 2zp+2zpq – 2y – 2yp – 2zq – 2zpq=0

2x(1+q) + 2zp(q – y) – 2y(1 – p) = 0

x(1+q)+ zp(q – y) = y(1 – p)

p(z – y) + q(x – z) = y – x

Which is the required solutions

2. **Form the PDE by eliminating arbitrary function,** $\phi(x^2 + y^2 + z^2, xyz) = 0$
   Solution:

   The given eq. is of the form $\emptyset(u, v) = 0$

   Where,

   $u = x^2+y^2+z^2$

   $v = xyz$

   $$u_x = 2x + 2z\frac{\partial z}{\partial x} \qquad v_x = y\left(x\frac{\partial z}{\partial x} + z\right)$$

   $$u_x = 2(x + zp) \qquad v_x = xyp + zy$$

$$u_y = 2y + 2z\frac{\partial z}{\partial y} \qquad v_y = x\left(y\frac{\partial z}{\partial y} + z\right)$$

$$u_y = 2(y + zq) \qquad v_y = xyq + zx$$

w. k. t,

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

$$\begin{vmatrix} 2x + 2zp & xyp + zy \\ 2y + 2zq & xyq + zx \end{vmatrix} = 0$$

$$(xyq + zx)(2x + 2zp) - (xyp + zy)(2y + 2zq) = 0$$

$$2x^2yq + 2zx^2 + 2xyzpq + 2z^2xp - 2zy^2 - 2xyzpq - 2z^2yq = 0$$

$$px(z^2 - y^2) + qy(x^2 - z^2) = z(y^2 - x^2)$$

Which is the required solutions

3. **Form the PDE by eliminating arbitrary function,** $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
Solution:

The given eq. is of the form $\emptyset(u, v) = 0$

Where,

$$u = x^2 + y^2 + z^2$$

$$v = z^2 - 2xy$$

$$u_x = 2x + 2z\frac{\partial z}{\partial x} \qquad v_x = 2z\frac{\partial z}{\partial x} - 2y$$

$$u_x = 2(x + zp) \qquad v_x = 2zp - 2y$$

$$u_y = 2y + 2z\frac{\partial z}{\partial y} \qquad v_y = 2z\frac{\partial z}{\partial y} - 2x$$

$$u_y = 2(y + zq) \qquad v_y = 2zq - 2x$$

w. k. t,

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

$$\begin{vmatrix} 2x+2zp & 2zp-2y \\ 2y+2zq & 2zq-2x \end{vmatrix} = 0$$

(2zq – 2x) (2x+ 2zp) – (2zp – 2y) (2y+2zq)=0

$4xzq - 4x^2 + 4z^2pq - 4xzp - 4yzp - 4z^2pq + 4y^2 + 4yzq = 0$

$-pz(x+y) - (x^2-y^2) + zq(x+y) = 0$

$(x+y)[zq - pz] = y^2 - x^2$

Which is the required solutions

METHOD 3:

If we eliminate 2 or more arbitrary functions from the given relation, we get 2nd or higher order PDE.

1. **Form the PDE by eliminating arbitrary function,** $z = yf(x) + xg(y)$

Solution:

Given: $z = yf(x) + xg(y)$ ……1

D.p.w.r.to x,

p =y f'(x)+ g(y) ……2

D.p.w.r.to y,

q = f(x) + x g'(y) ..……..3

D.p. eqn. 2 w.r. to x,

r = y f''(x) …….4

D.p. eqn.3 w.r. to y,

t = x g''(y) …….5

D. eqn.2 p.w.r.to y,

s = f'(x) + g'(y) …….6

eq. 2*x and eq. 3*y,

px=xy f'(x) + x g(y)

(+) qy=y f(x) +xy g'(y)

Px+ qy = xy[f'(x) + g'(y)] + x g(y) + y f(x) ……7

Sub. Eqn. 1 and Eqn.6 in Eqn.7,

Px+qy=xys+z

Which is the required solutions

2. Form the PDE by eliminating arbitrary function, z=x f(ax + by)+xg(ax + by)

Solution:

Z=x f(ax + by)+x g(ax + by) …….1

D.p.w.r.to x,

p = x f'(ax +by) (a) + f(ax+by) (1) + g'(ax+by) (a)

p= ax f'(ax +by) + f(ax+ by) +a g'(ax+by)

p= a[a f'(ax+by) + g'(ax+by)] + f(ax+by) ……..2

D.p.w.r.to y,

q = x f'(ax +by) (b) + g'(ax+by) (b)

q= b[x f'(ax+by) + g'(ax+by)] ……3

D .eqn. 2 p.w.r.to x,

r = a [x f'(ax+by) (a) + f'(ax+by) + g''(ax+by) (a)] + f'(ax+by)(a)

r = a^2x f'' (ax+by) + a f' (ax+by) + a^2 g'' (ax+by) + a f' (ax+by)

r = a^2 [x f'' (ax+by) + g' (ax+by)] + 2a f' (ax+by) ……..4

D. eqn. 3 p.w.r.to y,

t = b [x f'' (ax+by) (b) + g'' (ax+by) (b)]

t= b^2 [x f''(ax+by) + g'' (ax+by)] ……..5

D. eq. 2 p.w.r.to y,

s = a [bx f''(ax+by) + b g'' (ax+by)] + f'(ax+by) (b)

s= ab [x f" (ax+by) + g"(ax+by)] + b f'(ax+by) …….6

From eq. 4,

x f" (ax+by) + g" (ax+by) = t/b^2……7

Sub. Eqn. 7 in eqn. 4,

r = $\frac{a^2t}{b^2}$ + 2a f' (ax+by)

2a f' (ax+by) = r - $\frac{a^2t}{b^2}$

f '(ax+by) = $\frac{r}{2a} - \frac{at}{2b^2}$ ……..8

Sub. Eqn. 7 and eqn. 8 in eqn. 6,

s =$\frac{abt}{b^2}$ + b $\left[\frac{r}{2a} - \frac{at}{2b^2}\right]$ = $\frac{at}{b} + \frac{rb}{2a} - \frac{at}{2b}$

$$s=\frac{2a^2t + rb^2 - a^2t}{2ab}$$

2abs= $a^2t - rb^2$

Which is the required solutions

3. Form the PDE by eliminating arbitrary function, z= f(2x+y) + g(3x–y)

Solution:

Z=f(2x+y) + g(3x–y) ……1

D.p.w.r.to x,

p = f' (2x+y) (2) + g' (3x – y) (3)

p = 2f'(2x+y) + 3g'(3x – y) ……2

D.p.w.r. to y,

q = f'(2x+y) + g'(3x-y) (-1)

q= f'(2x+y) – g'(3x – y) ……3

D. eqn. 2 p.w.r.to x,

r = 2f"(2x+y)(2) + 3g"(3x–y)(3)

r= 4f"(2x+y) + 9g"(3x – y) …….4

D. eqn.3 p.w.r.to y,

$t = f''(2x+y) - g''(3x - y)(-1)$

$t = f''(2x+y) + g''(3x - y)$ …….5

D. eqn. 2 p.w.r.to y,

$s = 2f''(2x+y) + 3g''(3x - y)(-1)$

$s = 2f''(2x+y) - 3g''(3x-y)$

eqn. 5*3 + eqn. 6,

$$\begin{array}{rl} & 3f''(2x+y) + 3g''(3x - y) \\ (+) & 2f''(2x+y) - 3g''(3x - y) \\ \hline & 3t + s = 5f''(2x+y) \end{array}$$

$f''(2x+y) = \frac{3t+s}{5}$ ……..7

Sub. eq. 7 in eq. 5,

$t = \frac{3t+s}{5} + g''(3x - y)$

$t - \frac{3t+s}{5} = g''(3x-y)$

$\frac{2t-s}{5} = g''(3x - y)$ ……8

Sub. eqn. 7 and eqn. 8 in 4,

$r = 4\left(\frac{3t+s}{5}\right) + 9\left(\frac{2t-s}{5}\right)$

$5r = 12t + 4s + 18t - 9s = 30t - 5s$

$r = 6t - s$

Which is the required solutions

LAGRANGES LINEAR EQUATIONS

The equations of the form $pP+qQ = R$ is known as Lagrange's linear equation where P, Q, R is a function of x, y and z. To solve this equation, it is enough to solve the subsidiary equation or auxiliary equation

$$(i.e)\ \frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$$

TYPE 1: Method Of Grouping

1. **Solve** $px+qy = z.$

Solution:

The given equation is of the form $pP+qQ = R$

The subsidiary equations are, $(i.e)\ \frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$

Here P = x, Q = y, R = z

$$(i.e)\ \frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$$

Case 1:

Consider first two ratio,

i.e) $\frac{dx}{x}=\frac{dy}{y}$

Integrating both sides,

$$\int\frac{dx}{x}=\int\frac{dy}{y}$$

$$\log x = \log y + \log c_1$$

$$\log x = \log yc_1$$

$$\boxed{c_1=\frac{x}{y}}$$

Case 2:

Consider the second and third ratio

$$\frac{dy}{y} = \frac{dz}{z}$$

Integrating both sides

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y = \log z + \log C_2$$

$$\log\ y\ =\ \log\ zc_2$$

$$\boxed{c_2 = \frac{y}{z}}$$

Therefore the general solution of the equation is

$$\varphi(c_1, c_2) = 0$$

$$\boxed{\varphi\left(\frac{x}{y}, \frac{y}{z}\right) = 0}$$

2. **Solve** $pyz\ +\ qzx\ =\ xy.$

Solution:

The given equation is of the form $pP + qQ = R$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

Here $P\ = yz,\ Q = zx,\ R = xy$

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

Case 1:

Consider the first two ratio

$$\frac{dx}{yz} = \frac{dy}{zx}$$

$$x\,dx = y\,dy$$

Integrating both sides

$$\int x\,dx = \int y\,dy$$

$$\frac{x^2}{2} = \frac{y^2}{2} + c_1$$

$$c_1 = \frac{x^2 - y^2}{2}$$

$$\boxed{c_1 = x^2 - y^2}$$

Case 2:

Consider second and third ratio

$$\frac{dy}{zx} = \frac{dz}{xy}$$

$$y\,dy = z\,dz$$

Integrating both sides

$$\int y\,dy + \int z\,dz$$

$$\frac{y^2}{2} = \frac{z^2}{2} + c_2$$

$$\boxed{c_2 = y^2 - z^2}$$

The general solution is $\boxed{\varphi(x^2 - y^2, y^2 - z^2) = 0}$

3. Solve $\frac{y^2 z}{x} p + xzq = y^2$

Solution:

$$c\,P = \frac{y^2 z}{x}, \quad Q = xz, \quad R = yx$$

$$\therefore \quad \frac{x\,dx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{yz}$$

Case 1:

Consider the first two ratio

$$\frac{x\ dx}{y^2 z} = \frac{dy}{xz}$$

$$x^2\ dx\ =\ y^2\ dy$$

Integrating both sides

$$\int x^2\ dx = \int y^2\ dy$$

$$\frac{x^3}{3} = \frac{y^3}{3} + c_1$$

$$\boxed{c_1 = x^3 - y^3}$$

Case 2:

Consider first and third ratio

$$\frac{x\ dx}{y^2 z} = \frac{dz}{y^2}$$

$$x\ dx\ =\ z\ dz$$

Integrating both sides

$$\int x\ dx = \int z\ dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c_2$$

$$\boxed{c_2 = x^2 - z^2}$$

The general solution is $\boxed{\varphi(x^3 - y^3, x^2 - z^2) = 0}$

4. Solve $p - q\ =\ \log(x+y)$

Solution:

The given equation is of the form $pP + qQ = R$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

Here $P = 1,\quad Q = -1,\quad R = \log(x+y)$

$$\therefore\ dx = -dy = \frac{dz}{\log(x+y)}$$

Case 1:

Consider the first two ratio

$$dx = -dy$$

Integrating both sides

$$\int dx = -\int dy$$

$$x = -y + c_1$$

$$\boxed{c_1 = x + y}$$

Case 2:

Consider second and third ratio

$$\frac{dy}{-1} = \frac{dz}{\log(x+y)}$$

$$\log(x+y)\ dy = -1\ dz$$

Put $t = x + y$

$$\log t\ dy = -dz$$

Integrating both sides

$$\log t \int dy = -1 \int dz$$

$$\log t\ y = -z + c_2$$

$$\boxed{c_2 = y\ \log(x+y) + z}$$

The general solution is $\boxed{\varphi(x+y\ ,\ y\ \log(x+y)+z)}$

5. **Solve** $y^2 p - xyq = x(z - 2y)$

Solution:

The given equation is of the form $pP + qQ = R$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

Here $P = y^2,\ Q = xy,\ R = x(z-2y)$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

Case 1:

Consider the first two ratio

$$\frac{\mathrm{dx}}{\mathrm{y}^2} = \frac{\mathrm{dy}}{-xy}$$

$$x\ dx = -y\ dy$$

Integrating both sides

$$\int \mathrm{x\ dx} = -\int \mathrm{y\ dy}$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + c_1$$

$$c_1 = \frac{x^2 + y^2}{2}$$

$$\boxed{c_1 = x^2 + y^2}$$

Case 2:

Consider second and third ratio

$$\frac{\mathrm{dy}}{-xy} = \frac{dz}{x(z-2y)}$$

$$\frac{\mathrm{dy}}{-y} = \frac{dz}{(z-2y)}$$

$$\mathrm{z}\ dy - 2ydy = -\mathrm{y}\ dz$$

$$\mathrm{z}\ dy - 2ydy + \mathrm{y}\ dz = 0$$

$$d(yz) - 2ydy = 0$$

Integrating both sides

$$yz - y^2 = c_2$$

$$\boxed{c_2 = yz - y^2}$$

The general solution is $\boxed{\varphi(x^2 + y^2, yz - y^2) = 0}$

TYPE 2 METHOD OF MULTIPLIERS

1. Solve x(y – z)p+y(z – x)q=z(x – y)

Solution:

The given equation is of the form $pP + qQ = R$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \lambda$

Here x(y – z)p+y(z – x)q=z(x – y)

$$\therefore \frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} = \lambda$$

Case – 1:

Choose the multipliers l=1,m=1,n=1

$$\frac{dx + dy + dz}{xy - xz + yz - yx + zx - zy} = \lambda$$

Dx+dy+dz=0

Integrating,

$$\int dx + \int dy + \int dz = 0$$

$$\boxed{c_1 = x + y + z}$$

Case – 2:

Choose the multiplier $\ell = \frac{1}{x}, \quad m = \frac{1}{y}, \quad n = \frac{1}{z}$

$$\frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{y-z+z-x+x-y}=\lambda$$

$$\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}=\lambda$$

Integrating,

$$\int\frac{dx}{x}+\int\frac{dy}{y}+\int\frac{dz}{z}=\lambda$$

log x+log y+log z= c_2

log c_2 =log xyz

$$\boxed{c_2=xyz}$$

$\therefore$ The general solution is, $\boxed{\varphi(x+y+z,\ xyz)=0}$

2. Solve (mz – ny)p+(nx – lz)q=ly – mx.

Solution:

The equation is of the form pP+qQ=R

The subsidiary equations are $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}=\lambda$

Here P=(mz – ny),Q=(nx – lz),R=(ly – mx)

$$\therefore\ \frac{dx}{mx-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}=\lambda$$

Case-1:

Choose the multiplier x,y,z

$$\frac{x\ dx+y\ dy+z\ dz}{xmz-xny+ynx-ylz+zly-zmx}=\lambda$$

Integrating,

$$\int x\ dx+\int y\ dy+\int z\ dz=0$$

$$\frac{x^2}{2}=\frac{y^2}{2}=\frac{z^2}{2}=0$$

$$\boxed{c_1 = x^2 + y^2 + z^2}$$

Case-2:

Choose the multiplier l,m,n

$$\frac{l\ dx + m\ dy + n\ dz}{lmz - \ln y + mnx - mlz + nly - nmx} = \lambda$$

$$l\ dx + m\ dy + n\ dz = 0$$

Integrating,

$$\int l\ dx + \int m\ dy + \int n\ dz = 0$$

$$\boxed{c_2 = lx + my + nz}$$

$\therefore$ The general solution is, $\boxed{\varphi(x^2 + y^2 + z^2, lx + my + nz) = 0}$

3. **Solve** $(3z-4y)\frac{\partial z}{\partial x}+(4x-2z)\frac{\partial z}{\partial y}=(2y-3x)$

Solution:

W.k.t $\frac{\partial z}{\partial x} = p \quad and \quad \frac{\partial z}{\partial y} = q$

The equation can be written as $(3z-4y)p+(4x-2z)q=(2y-3x)$

The equation is of the form pP + qQ = R

The subsidiary equations are $\frac{\text{dx}}{\text{P}}=\frac{\text{dy}}{\text{Q}}=\frac{\text{dz}}{\text{R}} = \lambda$

Here P=(3z – 4y),Q=(4x – 2z),R=(2y – 3x)

$$\therefore \frac{dx}{3z-4y}=\frac{dy}{4x-2z}=\frac{dz}{2y-3x} = \lambda$$

Case-1:

Choose the multiplier x,y,z

$$\frac{x\ dx + y\ dy + z\ dz}{3xz - 4xy + 4yx - 2yz + 2zy - 3zx} = \lambda$$

Integrating,

$$\int x\ dx + \int y\ dy + \int z\ dz = 0$$

$$\frac{x^2}{2} = \frac{y^2}{2} = \frac{z^2}{2} = 0$$

$$\boxed{c_1 = x^2 + y^2 + z^2}$$

Case-2:

Choose the multiplier 2, 3, 4

$$\frac{2\ dx + 3\ dy + 4\ dz}{6z - 8y + 12x - 6z + 8y - 12x} = \lambda$$

$$2\,dx + 3\ dy + 4\ dz = 0$$

Integrating,

$$\int 2\ dx + \int 3\ dy + \int 4\ dz = 0$$

$$\boxed{c_2 = 2x + 3y + 4z}$$

$\therefore$ The general solution is, $\boxed{\varphi(x^2 + y^2 + z^2, 2x + 3y + 4z) = 0}$

4. **Solve** $x\left(y^2 + z^2\right)p + y\left(z^2 + x^2\right)q = z\left(y^2 - x^2\right)$

Solution:

The equation is of the form pP + qQ = R

The subsidiary equations are $\frac{\mathrm{dx}}{\mathrm{P}} = \frac{\mathrm{dy}}{\mathrm{Q}} = \frac{\mathrm{dz}}{\mathrm{R}} = \lambda$

Here P= $x\left(y^2 + z^2\right)$,Q= $y\left(z^2 + x^2\right)$,R= $z\left(y^2 - x^2\right)$

$$\therefore \quad \frac{dx}{x\left(y^2+z^2\right)}=\frac{dy}{y\left(z^2+x^2\right)}=\frac{dz}{z\left(y^2-x^2\right)}=\lambda$$

Case-1:

Choose the multiplier $\frac{-1}{x}, \frac{1}{y}, \frac{1}{z}$

$$\frac{-\frac{1}{x}\ dx+\frac{1}{y}\ dy+\frac{1}{z}\ dz}{-y^2-z^2+z^2+x^2+y^2-x^2}=\lambda$$

Integrating,

$$-\int\frac{1}{x}\ dx+\int\frac{1}{y}\ dy+\int\frac{1}{z}\ dz=0$$

$$-\log x+\log y+\log z=\log c_1$$

$$-\log x+\log\left(yz\right)=\log c_1$$

$$\log\left(\frac{yz}{x}\right)=\log c_1$$

$$\boxed{c_1=\left(\frac{yz}{x}\right)}$$

Case-2:

Choose the multiplier x, -y, z

$$\frac{x\,dx-y\ dy+z\ dz}{x^2y^2+x^2z^2-y^2z^2-y^2x^2+z^2y^2-z^2x^2}=\lambda$$

$$x\,dx-y\ dy+z\ dz=0$$

Integrating,

$$\int \mathrm{x}\ dx-\int \mathrm{y}\ dy+\int \mathrm{z}\ dz=0$$

$$\boxed{c_2=x^2-y^2+z^2}$$

$\therefore$ The general solution is, $\boxed{\varphi\left(\frac{yz}{x}, x^2 - y^2 + z^2\right) = 0}$

6. Solve $x\left(z^2 - y^2\right)p + y\left(x^2 - z^2\right)q = z\left(y^2 - x^2\right)$

Solution:

The equation is of the form pP + qQ = R

The subsidiary equations are $\frac{\text{dx}}{\text{P}} = \frac{\text{dy}}{\text{Q}} = \frac{\text{dz}}{\text{R}} = \lambda$

Here P= $x\left(z^2 - y^2\right)$,Q= $y\left(x^2 - z^2\right)$,R= $z\left(y^2 - x^2\right)$

$$\therefore \frac{dx}{x\left(z^2 - y^2\right)} = \frac{dy}{y\left(x^2 - z^2\right)} = \frac{dz}{z\left(y^2 - x^2\right)} = \lambda$$

Case-1:

Choose the multiplier $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$

$$\frac{\frac{1}{x}\ dx + \frac{1}{y}\ dy + \frac{1}{z}\ dz}{z^2 - y^2 + x^2 - z^2 + y^2 - x^2} = \lambda$$

Integrating,

$$\int \frac{1}{x}\ dx + \int \frac{1}{y}\ dy + \int \frac{1}{z}\ dz = 0$$

$$\log x + \log y + \log z = \log c_1$$

$$\log(xyz) = \log c_1$$

$$\boxed{c_1 = xyz}$$

Case-2:

Choose the multiplier x, y, z

$$\frac{x\,dx + y\ dy + z\ dz}{x^2z^2 - x^2y^2 + y^2x^2 - y^2z^2 + z^2y^2 - z^2x^2} = \lambda$$

$$x\,dx + y\,dy + z\,dz = 0$$

Integrating,

$$\int x\,dx + \int y\,dy + \int z\,dz = 0$$

$$\boxed{c_2 = x^2 + y^2 + z^2}$$

$\therefore$ The general solution is, $\boxed{\varphi\left(xyz,\ \ x^2 + y^2 + z^2\right) = 0}$

7. **Solve** $\left(\dfrac{y-z}{yz}\right)p + \left(\dfrac{z-x}{zx}\right)q = \left(\dfrac{x-y}{xy}\right)$

Solution:

Multiple given equation by xyz

We get $\quad x(y-z)p + y(z-x)q = z(x-y)$

The equation is of the form pP + qQ = R

The subsidiary equations are $\dfrac{\mathrm{dx}}{\mathrm{P}} = \dfrac{\mathrm{dy}}{\mathrm{Q}} = \dfrac{\mathrm{dz}}{\mathrm{R}} = \lambda$

Here P = $x(y-z)$, $\quad$ Q = $y(z-x)$, $\quad$ R = $z(x-y)$

$$\therefore\ \frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} = \lambda$$

Case - 1:

Choose the multiplier $\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$

$$\frac{\frac{1}{x}\,dx + \frac{1}{y}\,dy + \frac{1}{z}\,dz}{y - z + z - x + x - y} = \lambda$$

Integrating,

$$\int \frac{1}{x}\,dx + \int \frac{1}{y}\,dy + \int \frac{1}{z}\,dz = 0$$

$$\log x + \log y + \log z = \log c_1$$

$$\log(xyz) = \log c_1$$

$$\boxed{c_1 = xyz}$$

Case - 2:

Choose the multiplier 1, 1, 1

$$\frac{dx + dy + dz}{xy - xz + yz - xy + zx - zy} = \lambda$$

$$x\,dx + y\,dy + z\,dz = 0$$

Integrating,

$$\int dx + \int dy + \int dz = 0$$

$$\boxed{c_2 = x + y + z}$$

$\therefore$ The general solution is, $\boxed{\varphi\left(xyz,\quad x+y+z\right)=0}$

8. Solve $\left(z^2 - 2yz - y^2\right)p + \left(xy + zx\right)q = \left(xy - zx\right)$

Solution:

The equation is of the form pP + qQ = R

The subsidiary equations are $\frac{\text{dx}}{\text{P}} = \frac{\text{dy}}{\text{Q}} = \frac{\text{dz}}{\text{R}} = \lambda$

Here P $= \left(z^2 - 2yz - y^2\right)$, Q $= \left(xy + zx\right)$, R = $\left(xy - zx\right)$

$$\therefore \quad \frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{xy + zx} = \frac{dz}{xy - zx} = \lambda$$

Case - 1:

Consider the first two ratio

$$\frac{dy}{xy + zx} = \frac{dz}{xy - zx}$$

$$\frac{dy}{x\left(y+z\right)} = \frac{dz}{x\left(y-z\right)}$$

$$\left(y - z\right)dy = \left(y + z\right)dz$$

$$ydy - zdy = ydz + zdz$$

$$ydy - zdy - ydz - zdz = 0$$

$$ydy - d\left(yz\right) - zdz = 0$$

Integrating both sides

$$\int ydy - \int d\left(yz\right) - \int zdz = 0$$

$$\boxed{y^2 - yz - z^2 = c_1}$$

Case - 2:

Choose the multiplier x, y, z

$$\frac{x\,dx + y\ dy + z\ dz}{xz^2 - 2xyz - xy^2 + xy^2 + xyz + xyz - xz^2} = \lambda$$

$$x\,dx + y\ dy + z\ dz = 0$$

Integrating,

$$\int \mathrm{x}\ dx + \int \mathrm{y}\ dy + \int \mathrm{z}\ dz = 0$$

$$\boxed{c_2 = x^2 + y^2 + z^2}$$

$\therefore$ The general solution is, $\boxed{\varphi\left(y^2 - yz - z^2,\ \ x^2 + y^2 + z^2\right) = 0}$

9. **Solve** ($\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2$)p+2xyq=2zx.

Solution:

The equation is of the form pP+qQ=R

The subsidiary equations are $\frac{\mathrm{dx}}{\mathrm{P}} = \frac{\mathrm{dy}}{\mathrm{Q}} = \frac{\mathrm{dz}}{\mathrm{R}} = \lambda$

Here P= $\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2$,Q=2xy,R=2zx

Case – 1:

Consider second and third ratio,

$$\frac{dy}{zxy} = \frac{dz}{2zx}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

Integrating,

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

log y=log z+log c_1

$$c_1 = \frac{\log\ y}{\log\ z}$$

$$\boxed{c_1 = \frac{y}{z}}$$

Case – 2:

Using multiplier x,y,z we get each of the above ratio is equal to,

$$\frac{x\ dx + y\ dy + z\ dz}{x^3 - xy^2 - xz^2 + 2xy^2 + 2xz^2} = \lambda$$

$$\frac{x\ dx + y\ dy + z\ dz}{x^3 + xy^2 + xz^2} = \lambda$$

$$\frac{x\ dx + y\ dy + z\ dz}{x(x^2 + y^2 + z^2)} = \lambda$$

From first and third ratio,

$$\frac{x\ dx + y\ dy + z\ dz}{x(x^2 + y^2 + z^2)} = \frac{dz}{2xz}$$

$$\frac{2(x\ dx + y\ dy + z\ dz)}{x^2 + y^2 + z^2} = \frac{dz}{z}$$

Integrating,

$$\int \frac{2(x\ dx + y\ dy + z\ dz)}{x^2 + y^2 + z^2} = \int \frac{dz}{z}$$

Put $\quad x^2 + y^2 + z^2 = t$

2x dx+2y dy+2z dz=dt

$$\therefore \int \frac{dt}{t} = \int \frac{dz}{z}$$

log t=log z+log c_2

$$c_2 = \frac{t}{z}$$

$$\boxed{c_2 = \frac{x^2 + y^2 + z^2}{z}}$$

$\therefore$ The general solution is, $\boxed{\varphi\left(\frac{y}{z}, \frac{x^2+y^2+z^2}{z}\right)=0}$

9. **Solve z(x – y)=** $x^2 p - y^2 q$

Solution:

The equation is of the form pP+qQ=R

The subsidiary equations are $\frac{\text{dx}}{\text{P}} = \frac{\text{dy}}{\text{Q}} = \frac{\text{dz}}{\text{R}} = \lambda$

Here P= x^2 ,Q=- y^2 ,R=z(x – y)

Case – 1:

Consider first and second ratio,

$$\frac{dx}{x^2} = -\frac{dy}{y^2}$$

Integrating,

$$\int \frac{dx}{x^2} = -\int \frac{dy}{y^2}$$

$$-\frac{1}{x} = -\left(-\frac{1}{y}\right) + c_1$$

$$\boxed{c_1 = \frac{1}{x} + \frac{1}{y}}$$

Case – 2:

Consider, $\frac{dx+dy}{x+y} = \lambda$

$$\frac{dx+dy}{(x+y)(x-y)} = \lambda$$

Consider first and third ratio,

$$\frac{dx+dy}{(x+y)(x-y)} = \frac{dz}{z(x-y)}$$

$$\frac{dx+dy}{(x+y)}=\frac{dz}{z}$$

Integrating,

$$\int\frac{dx+dy}{(x+y)}=\int\frac{dz}{z}$$

$$\int\frac{d(x+y)}{(x+y)}=\int\frac{dz}{z}$$

log(x+y)=log z+log c_2

$$\boxed{c_2=\frac{x+y}{2}}$$

$\therefore$ The general solution is, $\boxed{\varphi\left(\frac{1}{x}+\frac{1}{y},\frac{x+y}{z}\right)=0}$

Standard Types

TYPE:1

The given equation is of the form F(p,q)=0

1. Solve $p^2+q^2=m^2$

   Solution:

   Given: $p^2+q^2=m^2$ ----------1

   Let us assume,

   $$z=ax+by+c\text{--------}2$$

   $D.p.w.r.to\ x\ \&\ y in$ --2

   $$\frac{\partial z}{\partial x}=a \qquad \text{and} \qquad \frac{\partial z}{\partial y}=b$$

   $$p=a \qquad \text{and} \qquad q=b$$

   Sub p and q values in ---------1

   $$a^2+b^2=m^2$$

$$b^2 = m^2 - a^2$$

$$b = \sqrt{m^2 - a^2}$$

Sub in -----2

$$z = ax + \sqrt{m^2 - a^2}\, y + c$$

This is the complete solution.

2. Solve $p^2 + q^2 = npq$

Solution:

The given equation is of the form F(p,q)=0

Given: $p^2 + q^2 = npq ------1$

Let us assume $z = ax + by + c --------2$

$D.p.w.r.to\, x \,\&\, y\ in --2$

$\frac{\partial z}{\partial x} = a$ and $\frac{\partial z}{\partial y} = b$

$p = a$ and $q = b$

Sub in (2)

Sub p and q values in----1

$$a^2 + b^2 = nab$$

$$b^2 - nab + a^2 = 0$$

$$b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{na \pm \sqrt{n^2 a^2 - 4a^2}}{2}$$

$$b = \frac{na \pm \sqrt{a^2 (n^2 - 4)}}{2}$$

$$b = \frac{a}{2}(n \pm \sqrt{n^2 - 4})$$

Sub in (2)

$$z = ax + a/2(n \pm \sqrt{n^2 - 4})y + c$$

This is the complete solution

3. **Solve** $\sqrt{p} + \sqrt{q} = 1$

Solution:

The given equation is of the form F(p,q)=0

Given:

Let us assume $z = ax + by + c - - - - - - - 2$

$D.p.w.r.to\, x \,\&\, y \;\; in - -2$

$\frac{\partial z}{\partial x} = a$ and $\frac{\partial z}{\partial y} = b$

$p = a$ and $q = b$

Sub in ---2

Sub p and q values in----1

$$\sqrt{a} + \sqrt{b = 1}$$

$$\sqrt{b} = 1 - \sqrt{a}$$

$$b = \left(1 - \sqrt{a}\right)^2$$

$$b = 1 + a - 2\sqrt{a}$$

Sub in (2)

$$z = ax + \left[1 + a - 2\sqrt{a}\right]y + c$$

This is the complete solution

4. **Solve p + q = pq**

Solution:

The given equation is of the form f(p,q)=0

Given:p+q=pq------1

Let us assume $z = ax + by + c - - - - - - - 2$

$D.p.w.r.to\, x \,\&\, y \;in --2$

$$\frac{\partial z}{\partial x} = a \qquad \text{and} \qquad \frac{\partial z}{\partial y} = b$$

$$p = a \qquad \text{and} \qquad q = b$$

Sub in ---2

a+b=ab

a+b – ab=0

a+b(1=a)=0

b(1 – a) =-a

$$b = \frac{a}{a-1}$$

Sub in ---2

$$z = ax + \left(\frac{a}{a-1}\right) y + c$$

This is the complete solution

TYPE:2(Clairaut's form equation)

The equation is of the form z=px+qy+f(p,q)

1. **Solve: z = px+qy+pq**
   Solution:

   The given equation is of the form clairaut's equation, z = px+qy+pq-------1

   Put p=a,q=b

   Z=ax+by+ab----------2

   This is the complete solution.

   To find singular solution:

   D,p. w.r.t 'a'

$$\frac{\partial z}{\partial a} = x + b$$

Equating to zero

$$\frac{\partial z}{\partial a} = 0 \Rightarrow x + b = 0$$

$$b = -x$$

D,p. w.r.t 'b'

$$\frac{\partial z}{\partial b} = y + a$$

Equating to zero

$$\frac{\partial z}{\partial b} = 0 \Rightarrow y + a = 0$$

$$a = -y$$

Sub a and b value in (2)

$$z = -xy - xy + xy$$

$$z = -xy$$

This is the singular solution.

2. Solve $z = px + qy + p^2q^2$

Solution:

Given $z = px + qy + p^2q^2 \text{-------} \rightarrow (1)$

The equation is of the form z=px+qy+f(p,q)

W. k. t p = a and q = b

$$z = ax + by + a^2b^2 \text{-------} \rightarrow (2)$$

This is the complete solution

To find singular solution:

D,p. w.r.t 'a'

$$\frac{\partial z}{\partial a} = x + 2(a)b^2$$

Equating to zero

$$x + 2(a)b^2 = 0$$

$$2ab^2 = -x$$

$$\boxed{x = -2ab^2}$$

D,p. w.r.t 'b'

$$\frac{\partial z}{\partial b} = y + a^2(2b)$$

Equating to zero

$$\frac{\partial z}{\partial b} = 0 \Rightarrow y + 2a^2b = 0$$

$$y + 2a^2b = 0$$

$$\boxed{y = -2a^2b}$$

Sub x and y values in eqn (2)

$$z = -2a^2b^2 - 2a^2b^2 + a^2b^2$$

$$z = -3a^2b^2$$

$$z^3 = -27a^6b^6 \quad -----(3)$$

Consider $x^2y^2 = 4a^2b^4 \times 4a^4b^2$

$$x^2y^2 = 16a^6b^6$$

$$a^6b^6 = \frac{1}{16}x^2y^2$$

Sub these values in (3)

$$z^3 = -\frac{27}{16}x^2y^2$$

$$16z^3 + 27x^2y^2 = 0$$

This is the singular solution.

3. **Solve** $z = px + qy + \sqrt{1 + p^2 + q^2}$

Solution:

Given $z = px + qy + \sqrt{1 + p^2 + q^2}$ **----------(1)**

The equation is of the form z=px+qy+f(p,q)

Put p=a,q=b

$$z = ax + by + \sqrt{1 + a^2 + b^2}$$ **----------(2)**

This is the complete solution.

To find singular solution:

D,p. w.r.t 'a'

$$\frac{\partial z}{\partial a} = x + 0 + \frac{1}{2\sqrt{1 + a^2 + b^2}}(2a)$$

Equating to zero

$$x + \frac{a}{\sqrt{1 + a^2 + b^2}} = 0$$

$$x = -\frac{a}{\sqrt{1 + a^2 + b^2}}$$

$$\boxed{a = -x\sqrt{1 + a^2 + b^2}}$$

D,p. w.r.t 'b'

$$\frac{\partial z}{\partial b} = y + 0 + \frac{1}{2\sqrt{1 + a^2 + b^2}}(2b)$$

Equating to zero

$$y + \frac{b}{\sqrt{1 + a^2 + b^2}} = 0$$

$$y = -\frac{b}{\sqrt{1 + a^2 + b^2}}$$

$$\boxed{b = -y\sqrt{1 + a^2 + b^2}}$$

Sub a and b value in (2)

$$z = -x^2\sqrt{1 + a^2 + b^2} - y^2\sqrt{1 + a^2 + b^2} + \sqrt{1 + a^2 + b^2}$$

$$\boxed{z = \left(1 - x^2 - y^2\right)\sqrt{1 + a^2 + b^2}} \text{----------}(3)$$

Consider $1 + a^2 + b^2 = 1 + x^2\left(1 + a^2 + b^2\right) + y^2\left(1 + a^2 + b^2\right)$

$$\left(1+a^2+b^2\right)-x^2\left(1+a^2+b^2\right)-y^2\left(1+a^2+b^2\right)=1$$

$$\left(1+a^2+b^2\right)\left(1-x^2-y^2\right)=1$$

$$\left(1+a^2+b^2\right)=\frac{1}{1-x^2-y^2}$$

$$\boxed{\sqrt{1+a^2+b^2}=\frac{1}{\sqrt{1-x^2-y^2}}}$$

Sub in (3)

$$z=\left(1-x^2-y^2\right)\frac{1}{\sqrt{1-x^2-y^2}}$$

$$z=\sqrt{1-x^2-y^2}$$

$$z^2=1-x^2-y^2$$

$$x^2+y^2+z^2=1$$

This is the singular solution.

Type – III

The equation is of the form F(z, p, q) = 0

1. **Solve:$z=p^2+q^2$**
   Solution:

$Z=p^2+q^2$----------(1)

The given equation is of the form, F(z,p,q)=0

Let z=f(x+ay) be the solution of equation (1)

Let u=x+ay

Z=f(u)

Differentiate partially with respect to x & y we get,

$$p=\frac{\partial z}{\partial u}\ \&\ q=a\times\frac{\partial z}{\partial u}$$

Substitute p and q value in equation (1),

$$z=\left(\frac{\partial z}{\partial u}\right)^2+\left(a\left(\frac{\partial z}{\partial u}\right)\right)^2$$

$$z=\left(\frac{\partial z}{\partial u}\right)^2+\left(a^2\left(\frac{\partial z}{\partial u}\right)\right)^2$$

$$z=\left(\frac{\partial z}{\partial u}\right)^2+(1+a^2)$$

Taking square root on both sides,

$$\sqrt{z}=\frac{\partial z}{\partial u}\sqrt{1+a^2}$$

$$\frac{\partial z}{\sqrt{z}}=\frac{\partial u}{\sqrt{1+a^2}}$$

$$\int\frac{dz}{\sqrt{z}}=\frac{1}{\sqrt{1+a^2}}\int du$$

$$\int z^{1/2}dz=\frac{1}{\sqrt{1+a^2}}u+c$$

$$\frac{z^{1/2}}{1/2}=\frac{u}{\sqrt{1+a^2}}+c$$

$$2\sqrt{z}=\frac{x+ay}{\sqrt{1+a^2}}+c\,.$$

It is a complete solution.

2. **Solve: $1+p^2+q^2$**
   Solution:

$Z^2=1+p^2+q^2$----------(1)

The given equation is of the form F(z,p,q)=0

Let z=f(x+ay) be the solution of equation (1),

Let u=x+ay

Z=f(u)

Differentiate equation (1) partially with respect to x & y we get,

$$p = \frac{\partial z}{\partial u} \,\&\, q = a \times \frac{\partial z}{\partial u}$$

Substitute p&q values in equation (1),

$$z^2 = 1 + \left(\frac{\partial z}{\partial u}\right)^2 + \left(a\left(\frac{\partial z}{\partial u}\right)\right)^2$$

$$z^2 = 1 + \left(\frac{\partial z}{\partial u}\right)^2 + a^2\left(\frac{\partial z}{\partial u}\right)^2$$

$$z^2 - 1 = \left(\frac{\partial z}{\partial u}\right)^2 + a^2\left(\frac{\partial z}{\partial u}\right)^2$$

$$z^2 - 1 = \left(\frac{\partial z}{\partial u}\right)^2 (1 + a^2)$$

Taking square root on both sides,

$$\sqrt{z^2 - 1} = \left(\frac{\partial z}{\partial u}\right)\sqrt{1 + a^2}$$

$$\frac{\partial z}{\sqrt{z^2 - 1}} = \frac{\partial u}{\sqrt{1 + a^2}}$$

$$\int \frac{\partial z}{\sqrt{z^2 - 1}} = \int \frac{\partial u}{\sqrt{1 + a^2}}$$

$$\int \frac{\partial z}{\sqrt{z^2 - 1}} = \frac{1}{\sqrt{1 + a^2}} \int du$$

$$\therefore \int \frac{\partial x}{\sqrt{x^2 - a^2}} = \frac{1}{a}\cosh^{-1}\left(x/a\right)$$

Here x=z ,a=1

$$\cosh^{-1} z = \frac{x + ay}{\sqrt{1^2 + a^2}} + c$$

It is a complete solution.

3. **Solve: $z^2(p^2+q^2+1)=1$**

   Solution:

This equation is of the form F(z,p,q)=0

$Z^2(p^2+q^2+1)=1$ ----------(1)

Let z=f(x+ay) be the solution of equation (1)

Let u= x+ay

Z=f(u)

Differentiate partially with respect to x & y we get,

$$p=\frac{\partial z}{\partial u} \quad \& \quad q=a\times\frac{\partial z}{\partial u}$$

Substitute p & q values in equation (1),

$$z^2\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(a\left(\frac{\partial z}{\partial u}\right)\right)^2+1\right]=1$$

$$z^2\left[\left(\frac{\partial z}{\partial u}\right)^2+a^2\left(\frac{\partial z}{\partial u}\right)^2+1\right]=1$$

$$\left(\frac{\partial z}{\partial u}\right)^2+a^2\left(\frac{\partial z}{\partial u}\right)^2+1=\frac{1}{z^2}$$

$$\left(\frac{\partial z}{\partial u}\right)^2(1+a^2)=\frac{1}{z^2}-1$$

$$\left(\frac{\partial z}{\partial u}\right)^2(1+a^2)=\frac{1-z^2}{z^2}$$

Taking square root on both sides,

$$\frac{\partial z}{\partial u}\sqrt{1+a^2}=\frac{\sqrt{1-z^2}}{z}\Rightarrow-\frac{\sqrt{z^2-1}}{z}$$

$$\frac{z.\partial z}{\sqrt{z^2-1}}=\frac{-\partial u}{\sqrt{1+a^2}}$$

$$\int\frac{z.\mathrm{dz}}{\sqrt{z^2-1}}=\frac{-1}{\sqrt{1+a^2}}\int du$$

Put $t=z^2-1$

dt =2zdz

$$zdz = \frac{dt}{2}$$

$$\int \frac{dt/2}{\sqrt{t}} = \frac{-1}{\sqrt{1+a^2}} \int du$$

$$\frac{1}{2}\int t^{-1/2} dt = \frac{-u}{\sqrt{1+a^2}} + c$$

$$\frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right] = \frac{-u}{\sqrt{1+a^2}} + c$$

$$t^{1/2} = \frac{-u}{\sqrt{1+a^2}} + c$$

$$\sqrt{z^2-1} + \frac{x+ay}{\sqrt{1+a^2}} = c$$

4. **Solve: p(1+q)=qz**

Solution:

This equation is of the form F(z, p,q)=0

P(1+q)=q^2----------(1)

Let z=f(x+ay) be the solution of equation (1),

Let u=x+ay

Z=f(u)

Differentiate partially with respect to x & y we get,

$$p = \frac{\partial z}{\partial u} \quad \& \quad q = a \times \frac{\partial z}{\partial u}$$

Substitute p& q values in equation in (1),

$$\left(\frac{\partial z}{\partial u}\right)\left[1+\left(a.\frac{\partial z}{\partial u}\right)\right] = \left(a.\frac{\partial z}{\partial u}\right)z$$

$$1+a.\frac{\partial z}{\partial u}=az$$

$$a.\frac{\partial z}{\partial u}=az-1$$

$$\frac{\partial z}{\partial u}=\frac{az-1}{a}$$

$$\frac{\partial z}{\partial u}=z-\frac{1}{a}$$

$$\frac{\partial z}{z-1/a}=\partial u$$

$$\int \frac{dz}{z-1/a}=\int \partial u$$

$$\log\left(z-1/a\right)=u+c$$

$$\log\left(z-1/a\right)=x+ay+c$$

TYPE 4:

The equation is of the form is $F_1(x, p) = F_2(y, q)$

1. **solve:** $p-q=x^2+y^2$

Solution

Given:

$$p-x^2=q+y^2$$

The given equation is of form $F_1(x,p)=F_2(y,q)$

$$p-x^2=q+y^2=a$$

$p-x^2=a$ $\qquad$ $q+y^2=a$

$p=a+x^2$ $\qquad$ $q=a-y^2$

w.k.t,

$$dz=pdx+qdy$$

$$dz = (a + x^2)dx + (a - y^2)dy$$

$$z = \int (a + x^2)dx + \int (a - y^2)dy$$

$$z = ax + (x^3 / 3) + ay - (y^3 / 3) + c$$

It is a complete solution.

2. Solve: $p + q = x + y$

Solution

Given: $p + q = x + y$

Rearranging equation,

$$p - x = y - q$$

The given equation is of form

$F_1(x,p)=F_2(y,q)$

$$p - x = y - q = a$$

$$p - x = a \quad y - q = a$$

$$p = a + x \quad q = y - a$$

w.k.t,

$$dz = pdx + qdy$$

$$\int dz = \int (a + x)dx + \int (y - a)dy$$

$$z = ax + (x^2 / 2) + (y^2 / 2) - ay + c$$

It is a complete solution.

3. Solve: $p^2 + q^2 = x^2 + y^2$

Solution:

Given: $p^2 + q^2 = x^2 + y^2$

Rearranging equation,

$$p^2 - x^2 = y^2 - q^2$$

The given equation is of the form

$F_1(x,p)=F_2(y,q)$

$$p^2 - x^2 = y^2 - q^2 = k^2$$

$$p^2 - x^2 = k^2 \qquad\qquad y^2 - q^2 = k^2$$

$$p^2 = x^2 + k^2 \qquad\qquad q^2 = y^2 - k^2$$

$$p = \sqrt{x^2 + k^2} \qquad\qquad q = \sqrt{y^2 - k^2}$$

w.k.t,

$$dz = pdx + qdy$$

$$\int dz = \int \sqrt{k^2 + x^2}\,dx + \int \sqrt{y^2 - k^2}\,dy$$

$$z = \left(\frac{x}{2}\right)\sqrt{x^2 + k^2} + \left(\frac{k^2}{2}\right)\sinh^{-1}\left(\frac{x}{k}\right) + \left(\frac{y}{2}\right)\sqrt{y^2 - k^2} + \left(\frac{k^2}{2}\right)\cosh^{-1}\left(\frac{y}{k}\right) + c$$

It is a complete solution.

4. Solve: $p + q = \sin x + \sin y$

Solution:

Given: $p + q = \sin x + \sin y$

Rearranging equation,

$$p - \sin x = \sin y - q = a$$

$$p - \sin x = a \qquad\qquad \sin y - q = a$$

$$p = a + \sin x \qquad\qquad q = \sin y - a$$

w.k.t,

$$dz = pdx + qdy$$

$$\int dz = \int (a + \sin x)dx + \int (\sin y - a)dy$$

$$z = ax - \cos x - \cos y - ay + c$$

It is a complete solution.

5. **Solve:** $q^2 - p = y - x$

Solution:

Given:

$$q^2 - p = y - x$$
$$q^2 - y = p - x$$

The given equation is of the form$F_1(x,p)=F_2(y,q)$

$$p - x = q^2 - y = a$$

$$p - x = a$$
$$p = a + x$$

$$q^2 - y = a$$
$$q^2 = a + y$$
$$q = \sqrt{a + y}$$

w.k.t,

$$dz = pdx + qdy$$

$$\int dz = \int (a + x)dx + \int \sqrt{a + y}dy$$

$$z = ax + \frac{x^2}{2} + \frac{2(a + y)^{(3/2)}}{3} + c$$

It is a complete solution.

6. **Solve:** $q = px + p^2$

Solution:

Given: $q = px + p^2$

Rearranging equation,

$$q = px + p^2 = k^2$$

$$q = k^2 \;\; px + p^2 = k^2$$

$$p^2 + px - k^2 = 0$$

$$p = \frac{-x \pm \sqrt{x^2 + 4k^2}}{2}$$

$$p = \frac{-x \pm \sqrt{x^2 + (2k)^2}}{2}$$

w.k.t $\quad dz = pdx + qdy$

$$\int dz = -\frac{1}{2}\int xdx \pm \frac{1}{2}\int \sqrt{x^2 + (2k)^2}\,dx + \int k^2 dy$$

$$Z = \left(-\frac{1}{2}\right)\left(\frac{x^2}{2}\right) \pm \frac{1}{2}\left[\frac{x}{2}\sqrt{x^2 + (2k)^2} + \frac{(2k)^2}{2}\sinh^{-1}\frac{x}{2k}\right] + k^2 y + c$$

$$Z = \frac{-x^2}{4} \pm \frac{x}{4}\sqrt{x^2 + (2k)^2} + \frac{(2k)^2}{4}\sinh^{-1}\left(\frac{x}{2k}\right) + k^2 y + c$$

$$Z = \frac{-x^2}{4} \pm \frac{x}{4}\sqrt{x^2 + (2k)^2} + k^2 \sinh^{-1}\left(\frac{x}{2k}\right) + k^2 y + c$$

It is a complete solution.

TYPE 5 :

The equation is of the form is

$$\mathbf{F}\left(x^m p,\ y^n q\right) = \mathbf{0} \longrightarrow \mathbf{1}$$

$$\mathbf{F}\left(z, x^m p, y^n q\right) = \mathbf{0} \longrightarrow \mathbf{2}$$

CASE 1:

If m $\neq$ 1 and n $\neq$ 1, then

put $X = x^{1-m}$

$$\frac{\partial X}{\partial x} = (1-m)\,x^{-m}$$

We know that,

$$\frac{\partial z}{\partial x} = p\ ;\ \frac{\partial z}{\partial y} = q$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial X} \times \frac{\partial X}{\partial x}$$

$$p = P(1-m)x^{-m}$$

$$px^{\ m} = P(1-m) \longrightarrow \mathbf{3}$$

Again put $Y = y^{1-n}$ **, then**

$$\frac{\partial Y}{\partial y} = (1-n)y^{-n}$$

We know that,

$$\frac{\partial z}{\partial y} = q \,;\, \frac{\partial z}{\partial Y} = Q$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial Y} \times \frac{\partial Y}{\partial y}$$

$$q = Q(1-n)y^{-n}$$

$$qy^{\,n} = Q(1-n) \longrightarrow \mathbf{4}$$

Substitute equation 3 and 4 in (1)

(1) $\Rightarrow$ **F(P,Q)=0**

(2) $\Rightarrow$ **F(Z,P,Q)=0**

Apply type - 1 and type - 3 for the above equations we get

CASE 2:

If m=-1 and n=1,then put Z= log z

Differentiating we get,

$$\frac{\partial Z}{\partial z} = \frac{1}{z}$$

We know that

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial z} \times \frac{\partial z}{\partial x}$$

$$P = \frac{1}{z}p \longrightarrow \mathbf{5}$$

Again If n=-1,similarly we get Q= $\frac{1}{z}$ **q** $\longrightarrow$ **6**

Substitute equation 5 and 6 equation in 1 and 2

(1) $\Rightarrow$ **F(P,Q)=0**

(2) $\Rightarrow$ F(Z,P,Q)=0

Apply type - 1 and type - 3 in the above equation

PROBLEMS:

1. **SOLVE :** $x^2 p + y^2 q = 0$

Solution:

Given $x^2 p + y^2 q = 0 \longrightarrow$ **1**

The equation is of the form $F\left(x^m p, y^n q\right) = 0$

Here m = 2 and n = 2, where m $\neq$ 1 and n $\neq$ 1

BY USING CASE 1:

Put $X = x^{1-m}$

$$X = x^{1-2}$$

$$X = x^{-1}$$

$$\frac{\partial X}{\partial x} = -x^{-2}$$

We know that,

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial X}\frac{\partial X}{\partial x}$$

$$p = P\left(-x^{-2}\right)$$

$$x^2 p = -P \longrightarrow \mathbf{2}$$

Again Put $Y = y^{1-n}$

$$Y = y^{1-2}$$

$$Y = y^{-1}$$

$$\frac{\partial Y}{\partial y} = -y^{-2}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial Y}\frac{\partial Y}{\partial y}$$

$$q = Q\left(-y^{-2}\right)$$

$$y^2 q = -Q \longrightarrow \mathbf{3}$$

Substitute 2 and 3 in equation 1

$$\mathbf{P + Q = 0} \longrightarrow \mathbf{4}$$

This equation is of the form F(P,Q)=0

Let Z=ax+by+c is the solution of equation 4

$$\frac{\partial Z}{\partial x} = a = P$$

$$\frac{\partial Z}{\partial y} = a = \mathbf{Q}$$

Substitute P and Q values in equation 4

a +b=0

a=-b

Substitute a=-b in equation 5

$\therefore$ **Z=ax – ay+c**

But $X = x^{-1}$

$$X = \frac{1}{x}$$

$$Y = y^{-1}$$

$$Y = \frac{1}{y}$$

$$\therefore \quad Z = \frac{a}{x} - \frac{a}{y} + c$$

2. **SOLVE:** $p^2 + x^2 y^2 q^2 = x^2 z^2$

Solution:

Given $p^2+x^2y^2q^2=x^2z^2 \longrightarrow$ **1**

Divide it by x^2

$$\frac{p^2}{x^2}+y^2q^2=z^2$$

The equation is of the form

$$\mathbf{F}\left(Z, x^m p, y^n q\right)=0$$

Here m=-1,n=1

$$\therefore \left(px^{-1}\right)+\left(yq\right)^2=z^2$$

CASE 1:

When m = - 1 and m $\neq$ 1

By using case 1,put

$$X=x^{1-m}$$

$X=x^2$ **,then**

$$\frac{\partial X}{\partial x}=2x$$

Now, $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial X}\times\frac{\partial X}{\partial x}$

$$p=P\times 2x$$

$$x^{-1}p=2P \quad ----\rightarrow 2$$

By using case 2,

Y=log y

$$\frac{\partial Y}{\partial y}=\frac{1}{y}$$

$$\frac{\partial Z}{\partial y}=\frac{\partial Z}{\partial Y}\times\frac{\partial Y}{\partial y}$$

$$q=\frac{Q}{y}$$

$$qy = Q \longrightarrow 3$$

Substitute 2 and 3 in 1

$$\therefore\ 4P^2 + Q^2 = Z^2$$

The above equation is the form of type 3

ie. F =(x+ay) and u=(x + ay)

z = f (u)

We know that,

$$P = \frac{dz}{du}; \qquad Q = a\frac{dz}{du}$$

Substitute in equation 4

$$4\left(\frac{dz}{du}\right)^2 + a^2\left(\frac{dz}{du}\right)^2 = z^2$$

$$\left(\frac{dz}{du}\right)^2\left(4 + a^2\right) = z^2$$

$$\frac{dz}{du} = \frac{z}{\sqrt{4 + a^2}}$$

$$\frac{dz}{z} = \frac{du}{\sqrt{4 + a^2}}$$

$$\int \frac{dz}{z} = \int \frac{du}{\sqrt{4 + a^2}}$$

$$\log z = \frac{1}{\sqrt{4 + a^2}}u + c$$

$$\log z = \frac{X + aY}{\sqrt{4 + a^2}} + c$$

TYPE 6

The equation is of the form

$$F\left(z^m p, z^m q\right) = 0 \longrightarrow 1$$

$$F_1\left(x, z^m p\right) = F_2\left(y, z^m q\right) \longrightarrow 2$$

Case 1:

If $m \neq -1$**, then**

$$Z = z^{m+1}$$

Partially differentiate with respect to z

$$\frac{\partial Z}{\partial z} = (m+1) z^m$$

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial z} \cdot \frac{\partial z}{\partial x}$$

$$P = (m+1) z^m p$$

$$z^m p = \frac{P}{m+1} \longrightarrow 3$$

Similarly

$$z^m q = \frac{Q}{m+1} \longrightarrow 4$$

Substitute 3 and 4 in equation 1 and 2

$$(1) \Rightarrow \quad F(P, Q) = 0$$

$$(2) \Rightarrow \quad F_1(x, P) = F_2(y, Q)$$

$\therefore$ **Apply type 1 and type 4**

CASE 2:

If $m = -1$

$$put \quad Z = \log z$$

$$\frac{\partial Z}{\partial z} = \frac{1}{z}$$

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial z} \times \frac{\partial z}{\partial x}$$

$$P = \frac{1}{z} p$$

$$P = \frac{p}{z}$$

similarly

$$Q = \frac{q}{z}$$

Substitute in 1 and 2

$$F(P,Q) = 0$$

$$F_1(x,p) = F_2(y,q)$$

PROBLEMS

1. **SOLVE:** $p^2 + q^2 = z^2(x^2 + y^2)$

Solution:

Given $p^2 + q^2 = z^2(x^2 + y^2)$

Divide throughout by z^2

$$\frac{p^2}{z^2} + \frac{q^2}{z^2} = x^2 + y^2$$

$$p^2 z^{-2} + q^2 z^{-2} = x^2 + y^2$$

$$(pz^{-1})^2 + (qz^{-1})^2 = x^2 + y^2$$

The above equation is of the form

$$F(z^m p,\ z^m q) = 0$$

Here m=-1,use case 2,

Put Z=logz

$$\frac{\partial Z}{\partial z} = \frac{1}{z}$$

$$\frac{\partial Z}{\partial X} = \frac{\partial Z}{\partial z} \times \frac{\partial z}{\partial x}$$

$$P = \frac{p}{z} \Rightarrow P = pz^{-1}$$

$$Q = \frac{q}{z} \Rightarrow Q = qz^{-1}$$

Substitute p and q in 1

$$P^2+Q^2=x^2+y^2$$

$$p^2-x^2=y^2-q^2=k^2$$

$$p^2-x^2=k^2, \quad y^2-q^2=k^2$$

$$p^2=k^2+x^2, \quad q^2=y^2-k^2$$

$$p=\sqrt{k^2+x^2} \qquad q=\sqrt{y^2+k^2}$$

We know that,

$$dz=Pdx+Qdy$$

$$dz=\sqrt{x^2+k^2}dx+\sqrt{y^2-k^2}dy$$

Integrating we get

$$z=\frac{x}{2}\sqrt{x^2+k^2}+\frac{k^2}{2}\sinh^{-1}\left(\frac{x}{k}\right)+\frac{y}{2}\sqrt{y^2-k^2}-\frac{k^2}{2}\cosh^{-1}\left(\frac{y}{k}\right)+c$$

2. SOLVE: $z^2\left(p^2+q^2\right)=x^2+y^2$

Solution:

Given $z^2\left(p^2+q^2\right)=x^2+y^2$

$$z^2p^2+z^2q^2=x^2+y^2$$

$$(zp)^2+(zq)^2=x^2+y^2 \longrightarrow 1$$

Here m = 1 $\neq -1$

Use case (1):

Put $Z=z^{m+1}$

$$z=z^2$$

$$\frac{\partial Z}{\partial z}=2z$$

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial z} \times \frac{\partial z}{\partial x}$$

$$P = 2zp$$

$$zp = \frac{p}{2} \longrightarrow 2$$

$$zq = \frac{q}{2} \longrightarrow 3$$

Substitute 2 and 3 in 1

$$\frac{p^2}{4} + \frac{q^2}{4} = x^2 + y^2$$

$$p^2 + q^2 = 4x^2 + 4y^2$$

$$p^2 - 4x^2 = 4y^2 - q^2 = k^2$$

$$p^2 - 4x^2 = k^2 \;\; and \;\; 4y^2 - q^2 = k^2$$

$$p^2 = 4x^2 + k^2 \;\; and \;\; q^2 = 4y^2 - k^2$$

$$p = \sqrt{4x^2 + k^2} \;\; and \;\; q = \sqrt{4y^2 - k^2}$$

We know that,

$$dz = Pdx + Qdy$$

$$dz = \sqrt{k^2 + 4x^2}\; dx + \sqrt{4y^2 - k^2}\; dy$$

Integrating we get

$$z = x\sqrt{4x^2 + k^2} + \frac{k^2}{2}\sinh^{-1}\left(\frac{2x}{k}\right) + y\sqrt{4y^2 - k^2} + \frac{k^2}{2}\cosh^{-1}\left(\frac{y}{k}\right) + c$$

Homogeneous Linear Partial Differential Equations

A homogeneous partial differential equation is of the form

$$\boxed{[a_0 D^n + a_1 D^{n-1} D^1 + \ldots + a_n (D')^n] z = F(x, y)} \longrightarrow (1)$$

Case (1):

The roots are real and distinct, then

$$\boxed{C.F = f_1(y+m_1x)+f_2(y+m_2x)}$$

Case (2):

The roots are real and equal, then

$$\boxed{C.F = f_1(y+m_1x)+xf_2(y+m_2x)}$$

Case (3):

The three roots are real and equal, then

$$\boxed{C.F = f_1(y+m_1x)+xf_2(y+m_2x)+x^2(y+mx)}$$

Case (4):

The roots are real and complex, then

$$\boxed{C.F = f_1(y+m_1x)+f_2(y+m_2x)}$$

Example $m_1 = 2 \;\; and \;\; m_2 = 1 \pm i$, then

$$\boxed{C.F = f_1(y+2x)+f_2(y+(1\pm i)x)}$$

TYPE 1: [Exponential]

1. **Solve** $\frac{\partial^2 Z}{\partial x^2}+(-3)\frac{\partial^2 Z}{\partial x \partial y}+2\frac{\partial^2 Z}{\partial y^2}=e^{3x+4y}$

Solution:

$$Put \;\; \frac{\partial}{\partial x}=D, \;\; \frac{\partial}{\partial y}=D'$$

The above equation becomes $[D^2-3DD'+2D'^2]Z \;=\; e^{3x+4y}$

To Find CF:

$$Put \; D=m \; and \; D'=1$$

$$m^2-3m+2=0$$

$$m=1, \;\; 2$$

$$\therefore \boxed{C.F = f_1(y_1 + x) + f_2(y_2 + 2x)}$$

To find PI:

$$PI = \frac{1}{D^2 - 3DD' + 2D'^2} e^{3x+4y}$$

$$Put\ D = 3,\quad D' = 4$$

$$PI = \frac{1}{9 - 3(3)(4) + 2(16)} e^{3x+4y}$$

$$PI = \frac{1}{9 - 36 + 32} e^{3x+4y}$$

$$\text{PI} = \frac{1}{5} e^{3x+4y}$$

The complete solution is CF + PI

$$z = f_1(y_1 + x) + f_2(y_2 + 2x) + \frac{1}{5} e^{3x+4y}$$

2. Solve $\frac{\partial^2 Z}{\partial x^2} + (-5)\frac{\partial^2 Z}{\partial x \partial y} + 6\frac{\partial^2 Z}{\partial y^2} = e^{x+y}$

Solution:

$$Put\ \frac{\partial}{\partial x} = D,\quad \frac{\partial}{\partial y} = D'$$

The above equation becomes $[D^2 - 5DD' + 6D'^2]Z = e^{x+y}$

To Find CF:

$$Put\ D = m\ and\ D' = 1$$
$$m^2 - 5m + 6 = 0$$
$$m = 3,\quad 2$$

$$\therefore \boxed{C.F = f_1(y_1 + 2x) + f_2(y_2 + 3x)}$$

To find PI:

$$PI = \frac{1}{D^2 - 5DD' + 6D'^2} e^{x+y}$$

$$Put\ D = 1,\quad D' = 1$$

$$PI = \frac{1}{1-5+6} e^{x+y}$$

$$\text{PI} = \frac{1}{2} e^{x+y}$$

The complete solution is CF + PI

$$z = f_1(y_1 + 2x) + f_2(y_2 + 3x) + \frac{1}{2} e^{x+y}$$

3. Solve $(D^3 - 7DD'^2 - 6D'^3)Z = e^{3x+y}$

Solution:

To find CF:

$$Put\ D = m\ and\ D' = 1$$

$$m^3 - 7m - 6 = 0$$

$$Put\ m = -1$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & 0 & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$m = -1 \quad and \quad (m^2 - m - 6) = 0$$

$$m = -1\ and \quad (m\text{-}3)\ (m+2) = 0$$

$$m = -1,\ -2,\ \ 3$$

$$\therefore \boxed{C.F = f_1(y_1 - x) + f_2(y_2 - 2x) + f_3(y + 3x)}$$

To find PI:

$$PI = \frac{1}{D^2 - 7DD' - 6D'^2} e^{3x+y}$$

$$Put\ D = 3,\ D' = 1$$

$$PI = \frac{1}{27 - 7(3) - 6} e^{3x+y}$$

$$PI = \frac{1}{27 - 21 - 6} e^{3x+y}$$

Dr = 0, then

$$PI = \frac{x}{3D^2 - 7D'^2 + 0} e^{3x+y}$$

$$Put\ D = 3,\quad D' = 1$$

$$PI = \frac{x}{3(9)-7} e^{3x+y}$$

$$\text{PI} = \frac{x}{27-7} e^{3x+y}$$

$$\text{PI} = \frac{x}{20} e^{3x+y}$$

The complete solution is CF + PI

$$z = f_1(y_1 - x) + f_2(y_2 - 2x) + f_3(y_3 + 3x) + \frac{x}{20} e^{3x+y}$$

4. Solve $(D^3 - 3DD'^2 + 2D'^3)Z = e^{2x-y} + e^{x+y}$

Solution:

To find CF:

$Put\ D = m\ and\ D' = 1$

$m^3 - 3m + 2 = 0$

$Put\ m = 1$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$m = 1\ and\ (m^2 + m - 2) = 0$

$m = 1\ and\ (m\text{-}1)\ (m+2) = 0$

$m = 1,\ -2,\ 1$

$\therefore \boxed{C.F = f_1(y_1 + x) + xf_2(y_2 + x) + f_3(y - 2x)}$

To find PI:

$$PI_1 = \frac{1}{D^3 - 3DD'^2 + 2D'^3} e^{2x-y}$$

$Put\ D = 2,\quad D' = \text{-}1$

$$PI_1 = \frac{1}{8-6-2} e^{2x-y}$$

$$\text{PI}_1 = \frac{x}{3D^2 - 3D'^2 + 0} e^{2x-y}$$

$Put\ D = 2,\ D' = \text{-}1$

$$PI_1 = \frac{x}{3(4)-3} e^{2x-y}$$

$$PI_1 = \frac{x}{12-3} e^{2x-y}$$

$$\mathrm{PI}_1 = \frac{x}{9} e^{2x-y}$$

Now $$PI_2 = \frac{1}{D^3 - 3DD'^2 + 2D'^3} e^{x+y}$$

$$Put\ D = 1,\ \ D' = 1$$

$$PI_2 = \frac{1}{1-3+2} e^{x+y}$$

Dr = 0, then

$$PI_2 = \frac{x}{3D^2 - 3D'^2 + 0} e^{x+y}$$

$$Put\ D = 1,\ D' = 1$$

$$PI_2 = \frac{x}{3(1)-3} e^{x+y}$$

Dr = 0, then

$$PI_2 \ = \frac{x^2}{6D} e^{x+y}$$

$$PI_2 = \frac{x^2}{6} e^{x+y}$$

The complete solution is $z = C.F + PI_1 + PI_2$

$$z = f_1(y_1 + x) + xf_2(y_2 + x) + f_3(y_3 - 2x) + \frac{x}{9} e^{2x-y} + \frac{x^2}{6} e^{x+y}$$

TYPE II [Trigonometry]

$$f(D^2, DD', D'^2) = \sin(ax+by)\ or\ \cos(ax+by)$$

$$P.I = \frac{1}{f(D^2, DD', D'^2)}\ \sin(ax+by)\ or\ \cos(ax+by)$$

$$where\ D^2 = -a^2$$

$$D'^2 = -b^2$$

$$DD' = -ab$$

$$\therefore\ P.I = \frac{1}{f(-a^2,-b^2,-ab)}\ \sin(ax+by)\ or\ \cos(ax+by)$$

NOTE:

$$D = \frac{\partial}{\partial x}, \qquad D' = \frac{\partial}{\partial y}$$

$$\frac{1}{D} = \int dx, \quad \frac{1}{D'} = \int dy$$

1. **Solve** $(D^2+3DD'+2D'^2)\,z = \sin(x+3y)$

Solution:

To find CF:

Put D = m, D' =1

The auxiliary equation is,

$$m^2+3m+2=0$$

$$(m+2)(m+1) = 0$$

$$m = -2, -1$$

$$CF = f_1(y-x) + f_2(y-2x)$$

To find PI:

$$P.I = \frac{1}{D^2+3DD'+2D'^2)}\ \sin(x+3y)$$

Put $D^2 = -a^2 = -1$

$DD' = -ab = -3$

$D'^2 = -b^2 = -9$

$$PI = \frac{1}{-1-9-18}\ \sin(x+3y)$$

$$PI = -\frac{1}{28}\ \sin(x+3y)$$

The complete solution is $z = f_1(y-x) + f_2(y-2x) - \frac{1}{28}\ \sin(x+3y)$

2. Solve $(D^3 - 4D^2D' + 4DD'^2)z = 6\sin(3x+6y)$

Solution:

To find CF:

Put D = m, $D' = 1$

The auxiliary equation is,

$$m^3 - 4m^2 + 4m = 0$$

$$m(m^2 - 4m + 4) = 0$$

$$m = 0,\ m^2 - 4m + 4 = 0$$

$$m = 0 \text{ and } (m-2)(m-2) = 0$$

$$\therefore m = 0, 2, 2$$

$$CF = f_1(y) + f_2(y+2x) + xf_3(y+2x)$$

To find PI:

$$PI = \frac{1}{D^3 - 4D^2D' + 4DD'^2}\ 6\sin(3x+6y)$$

Put $D^2 = -a^2 = -9$

$D'^2 = -b^2 = -36$

$DD' = -ab = -18$

$$PI = \frac{1}{D(D^2 - 4DD' + 4D'^2)}\ 6\sin(3x+6y)$$

$$PI = \frac{1}{D(-9+72-144)}\ 6\sin(3x+6y)$$

$$PI = -\frac{1}{81D}\ 6\sin(3x+6y)$$

$$PI = -\frac{2}{27D}\ \sin(3x+6y)$$

$$PI = \frac{2}{81}\ \cos(3x+6y)$$

The complete solution is

$$\therefore z = f_1(y) + f_2(y+2x) + xf_3(y+2x) + \frac{2}{81}\cos(3x+6y)$$

3. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \sin 2x \sin 3y$

Solution:

Put $\frac{\partial}{\partial x} = D, \quad \frac{\partial}{\partial y} = D'$

$$z(D^2 - D'^2) = \sin 2x \ \sin 3y$$

$$z(D^2 - D'^2) = \frac{\cos(2x-3y) - \cos(2x+3y)}{2}$$

To find CF:

Put D = m, D' = 1

$$m^2 - 1 = 0$$

(m + 1) (m – 1) = 0

m = - 1, 1

$$\therefore CF = f_1(y-x) + f_2(y+x)$$

To find PI:

$$PI = \frac{1}{D^2 - D'^2}\left[\frac{\cos(2x-3y) - \cos(2x+3y)}{2}\right]$$

Now $PI_1 = \frac{1}{D^2 - D'^2}\left[\frac{\cos(2x-3y)}{2}\right]$

Put $D^2 = -a^2 = -4$
$D'^2 = -b^2 = -9$

$$\mathrm{PI}_1 = \frac{1}{2} \times \frac{1}{-4+9} \quad \cos(2x-3y)$$

$$PI_1 = \frac{\cos(2x-3y)}{10}$$

Now $PI_2 = -\frac{1}{D^2 - D'^2} \left[\frac{\cos(2x+3y)}{2} \right]$

$$PI_2 = -\frac{\cos(2x+3y)}{10}$$

The complete solution is $z = C.F + PI_1 + PI_2$

$$\therefore z = f_1(y-x) + f_2(y+x) + \frac{\cos(2x-3y)}{10} - \frac{\cos(2x+3y)}{10}$$

TYPE 3

The equation is of the form $f(D, D') = x^n y^n$

$(1+x)^{-1} = 1 - x + x^2 - ....$

$(1-x)^{-1} = 1 + x + x^2 + ...$

Problems

1. **Solve** $\left[D^2 + 3DD' + 2D'^2 \right] z = x + y$

   Solution:

   To find CF:

   $$Put\ D = m \quad and \quad D' = 1$$

   $$m^2 + 3m + 2 = 0$$

   $$(m+2)(m+1) = 0$$

   $$m = -1,\ -2$$

   $$\boxed{C.F = f_1(y-x) + f_2(y-2x)}$$

To find PI:

$$PI = \frac{1}{D^2 + 3DD' + 2D'^2} x + y$$

$$= \frac{1}{D^2\left[1 + \frac{3DD'}{D^2} + \frac{2D'^2}{D^2}\right]}(x+y)$$

$$= \frac{1}{D^2\left[1 + \left(\frac{3DD'}{D^2} + \frac{2D'^2}{D^2}\right)\right]}(x+y)$$

$$= \frac{1}{D^2}\left[1 + \left(\frac{3DD'}{D^2} + \frac{2D'^2}{D^2}\right)\right]^{-1}(x+y)$$

$$= \frac{1}{D^2}\left[1 - \left(\frac{3DD'}{D^2} + \frac{2D'^2}{D^2}\right)\right](x+y)$$

$$= \frac{1}{D^2}\left[1 - \frac{3DD'}{D^2} - \frac{2D'^2}{D^2}\right](x+y)$$

$$= \frac{1}{D^2}\left[(x+y) - \frac{3D'}{D}(x+y)\right]$$

$$= \frac{1}{D^2}\left[(x+y) - \frac{3}{D}(1)\right]$$

$$= \frac{1}{D^2}\left[(x+y) - 3x\right]$$

$$= \frac{1}{D^2}\left[y - 2x\right]$$

$$= \frac{1}{D}\left[yx - x^2\right]$$

$$= \left[\frac{yx^2}{2} - \frac{x^3}{3}\right]$$

The solution is z = CF + PI

$$\boxed{z = f_1(y-x) + f_2(y-2x) + \frac{yx^2}{2} - \frac{x^3}{3}}$$

2. Solve $\left[D^3 - 7DD'^2 - 6D'^2\right]z = x^2y + \sin(x+2y)$

Solution:

To find CF:

$$Put\ D = m \quad and \quad D' = 1$$

$$m^3 - 7m^2 - 6 = 0$$

$$m = -1 \ and \quad m^2 - m - 6 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & 0 & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$m = -1 \ and \quad (m-3)(m+2) = 0$$

$$m = -1, -2, 3$$

$$\therefore \boxed{C.F = f_1(y-x) + f_2(y-2x) + f_3(y+3x)}$$

To Find PI₁:

$$PI = \frac{1}{D^3 - 7DD'^2 - 6D'^3} x^2 y$$

$$= \frac{1}{D^3\left[1 - \left(\frac{7DD'^2}{D^3} - \frac{6D'^3}{D^3}\right)\right]}(x^2y)$$

$$= \frac{1}{D^3}\left[1 - \left(\frac{7D'^2}{D^2} + \frac{6D'^3}{D^3}\right)\right]^{-1}(x^2y)$$

$$= \frac{1}{D^3}\left[1 + \frac{7D'^2}{D^2} + \frac{6D'^3}{D^3}\right](x^2y)$$

$$= \frac{1}{D^3}\left[(1)x^2y\right] \quad \sin ce \ \ D'^2 \ \ and \ \ D'^3 \ \ are \ \ zero$$

$$= \frac{x^2 y}{D^3}$$

$$= \iiint x^2 y \; dx \; dx \; dx$$

$$= \iint \frac{x^3 y}{3} dx \; dx$$

$$= \int \frac{x^4 y}{12} dx$$

$$PI_1 = \frac{x^5 y}{60}$$

To find PI_2:

$$PI_2 = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \sin(x+2y)$$

$$= \frac{1}{D(D^2) - 7DD'(D') - 6D'^2(D')} \sin(x+2y)$$

$$Put \; D^2 = -1, \quad DD' = -2 \quad and \quad D'^2 = -4$$

$$= \frac{1}{D(-1) - 7(-2)D' - 6(-4)D'} \sin(x+2y)$$

$$= \frac{1}{-D + 14D' + 24D'} \sin(x+2y)$$

$$= \frac{1}{38D' - D} \sin(x+2y)$$

$$= \frac{(38D' + D)}{(38D' - D)(38D' + D)} \sin(x+2y)$$

$$= \frac{(38D' + D)}{1444D'^2 - D^2} \sin(x+2y)$$

$$Put \; \mathrm{D}^2 = a^2 = -1 \; and \; D'^2 = b^2 = -4$$

$$= \frac{38D'(\sin(x+2y)) + D(\sin(x+2y))}{1444(-4) + 1}$$

$$= \frac{38\cos(x+2y)(2)+\cos(x+2y)(1)}{-5776+1}$$

$$= \frac{76\cos(x+2y)+\cos(x+2y)}{-5775}$$

$$= -\frac{77\cos(x+2y)}{5775}$$

The general solution is $z = CF + PI_1 + PI_2$

$$z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) + \frac{x^5 y}{60} - \frac{77\cos(x+2y)}{5775}$$

3. Solve $\left[D^2 - 4DD' + 4D'^2\right]z = xy + e^{2x+y}$

Solution:

To Find CF:

$Put\ D = m$ and $D' = 1$

$m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m = 2,\ 2$

$$\boxed{C.F = f_1(y+2x) + xf_2(y+2x)}$$

To Find PI₁:

$$PI_1 = \frac{1}{D^2 - 4DD' + 4D'^2}xy$$

$$= \frac{1}{D^2\left[1 - \frac{4D'}{D} + \frac{4D'^2}{D^2}\right]}(xy)$$

$$= \frac{1}{D^2\left[1 - \left(\frac{4D'}{D} - \frac{4D'^2}{D^2}\right)\right]}(xy)$$

$$= \frac{1}{D^2}\left[1 - \left(\frac{4D'}{D} - \frac{4D'^2}{D^2}\right)\right]^{-1}(x+y)$$

$$= \frac{1}{D^2}\left[1 + \frac{4D'}{D} - \frac{4D'^2}{D^2}\right](xy)$$

$$= \frac{1}{D^2}\left[xy + \frac{4D'}{D}(xy)\right]$$

$$= \frac{1}{D^2}\left[xy + \frac{4x}{D}\right]$$

$$= \frac{1}{D^2}\left[xy + 4\int xdx\right]$$

$$= \frac{1}{D^2}\left[xy + \frac{4x^2}{2}\right]$$

$$= \frac{1}{D^2}\left[xy + 2x^2\right]$$

$$= \iint xy + 2x^2 dx\ dx$$

$$= \int\left[\frac{x^2 y}{2} + \frac{2x^3}{3}\right]dx$$

$$PI_1 = \frac{x^3 y}{6} + \frac{2x^4}{12}$$

To Find PI_2 **:**

$$PI_2 = \frac{1}{D^2 - 4DD' + 4D'^2} e^{2x+y}$$

$$Put\ D = 2, \quad and \quad D' = 1$$

$$= \frac{1}{4 - 8 + 4} e^{2x+y}$$

$$= \frac{x}{2D - 4D'} e^{2x+y}$$

$$= \frac{x}{4 - 4} e^{2x+y}$$

$$= \frac{x^2}{2} e^{2x+y}$$

The general solution is $z = CF + PI_1 + PI_2$

$$z = f_1(y+2x) + xf_2(y+2x) + \frac{x^3 y}{6} + \frac{2x^4}{12} + \frac{x^2}{2} e^{2x+y}$$

TYPE 4

1. **Solve** $\left[D^2 - 2DD' + D'^2\right] z = x^2 y^2 e^{x+y}$

Solution:

To find CF:

$Put\ D = m$ and $D' = 1$

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1, 1$

$$\boxed{C.F = f_1(y+x) + xf_2(y+x)}$$

To Find PI:

$$PI = \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 e^{x+y}$$

$$PI = \frac{1}{D^2 - 2DD' + D'^2} e^{x+y} x^2 y^2$$

$Put \quad D = D+1, \quad D' = D'+1$

$$PI = e^{x+y} \frac{1}{(D+1)^2 - 2(D+1)(D'+1) + (D'+1)^2} x^2 y^2$$

$$PI = e^{x+y} \frac{1}{D^2 + 2D + 1 - 2(DD' + D + D' + 1) + D'^2 + 2D' + 1} x^2 y^2$$

$$PI = e^{x+y} \frac{1}{D^2 + 2D + 1 - 2DD' - 2D - 2D' - 2 + D'^2 + 2D' + 1} x^2 y^2$$

$$PI = e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2$$

$$PI = e^{x+y}\frac{1}{D^2\left[1-\frac{2D'}{D}+\frac{D'^2}{D^2}\right]}x^2y^2$$

$$PI = e^{x+y}\frac{1}{D^2\left[1-\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right]}x^2y^2$$

$$PI = e^{x+y}\frac{1}{D^2}\left[1-\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right]^{-1}x^2y^2$$

$$PI = e^{x+y}\frac{1}{D^2}\left[1+\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)+\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)^2\right]x^2y^2$$

$$PI = e^{x+y}\frac{1}{D^2}\left[1+\frac{2D'}{D}-\frac{D'^2}{D^2}+\frac{4D'^2}{D^2}\right]x^2y^2$$

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{2D'}{D}\left(x^2y^2\right)-\frac{D'^2}{D^2}\left(x^2y^2\right)+\frac{4D'^2}{D^2}\left(x^2y^2\right)\right]$$

$$but \quad D'\left(x^2y^2\right)=2x^2y \textbf{ and } D'^2\left(x^2y^2\right)=2x^2$$

Sub in above eqn

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{2}{D}2x^2y-\frac{1}{D^2}2x^2+\frac{4}{D^2}2x^2\right]$$

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{4}{D}x^2y-\frac{2}{D^2}x^2+\frac{8}{D^2}x^2\right]$$

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{4x^3y}{3}-\frac{2x^3}{3D}+\frac{8x^3}{3D}\right]$$

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{4x^3y}{3}+\frac{6x^3}{3D}\right]$$

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{4x^3y}{3}+\frac{2x^4}{4}\right]$$

$$PI = e^{x+y}\frac{1}{D^2}\left[x^2y^2 + \frac{4x^3y}{3} + \frac{x^4}{2}\right]$$

$$PI = e^{x+y}\frac{1}{D}\left[\frac{x^3y^2}{3} + \frac{4x^4y}{3\times 4} + \frac{x^5}{10}\right]$$

$$PI = e^{x+y}\left[\frac{x^4y^2}{12} + \frac{x^5y}{15} + \frac{x^6}{60}\right]$$

The complete solution is z = CF + PI

$$\boxed{z = f_1(y+x) + xf_2(y+x) + e^{x+y}\left[\frac{x^4y^2}{12} + \frac{x^5y}{15} + \frac{x^6}{60}\right]}$$

2. **Solve** $\left[D^2 - DD' - 2D'^2\right]z = (y-1)e^x$

To find CF:

$Put\ D = m$ and $D' = 1$

$$m^2 - m - 2 = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1,\ 2$$

$$\boxed{C.F = f_1(y-x) + f_2(y+2x)}$$

To Find PI:

$$PI = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$

$Put \quad D = D+1, \quad D' = D'$

$$PI = e^x\frac{1}{(D+1)^2 - (D+1)D' - 2D'^2}(y-1)$$

$$PI = e^x\frac{1}{D^2 + 2D + 1 - DD' - D' - 2D'^2}(y-1)$$

$$PI = e^x\frac{1}{1 + D^2 + 2D - D' - DD' - 2D'^2}(y-1)$$

$$PI = e^x\left[1 + \left(D^2 + 2D - D' - DD' - 2D'^2\right)\right]^{-1}(y-1)$$

$$PI = e^x\left[1-\left(D^2+2D-D'-DD'-2D'^2\right)\right](y-1)$$

$$PI = e^x\left[1-D^2-2D+D'+DD'+2D'^2\right](y-1)$$

$$PI = e^x\left[(y-1)-D^2(y-1)-2D(y-1)+D'(y-1)+DD'(y-1)+2D'^2(y-1)\right]$$

$$PI = e^x\left[y-1-0-2(0)+(1)+0+0\right]$$

$$PI = e^x\left[y-1+1\right]$$

$$PI = ye^x$$

The complete solution is z = CF + PI

$$\boxed{z = f_1(y-x)+f_2(y+2x)+ye^x}$$

Non – Homogenous Linear Equation

Case 1:

(D – mD' – c)=z

CF: $z=e^{cx}f(y+mx)$

(D – mD – c)z=0

CF: $z=e^{cy}f(x+my)$

1. **Solve** $(D^2-D'^2+3D'-3D)z=0$

Solution:

$$[(D+D')(D-D')-3(D-D')]z=0$$

$$[(D-D')(D+D'-3)]z=0$$

$$[(D-D'+0)(D-(-D')-3)]=0$$

$$m_1=1;\quad m_2=-1$$

$$c_1=0;\quad c_2=3$$

$$z=e^0 f(y+x)+e^{3x}f(y-x)$$

$$z=f(y+x)+e^{3x}f(y-x)$$

2. **Solve** $(D^2+2DD'+D'^2-2D-2D')z=0$

Solution:

$$[(D+D')^2-2D-2D']z=0$$

$$[(D+D')^2-2(D+D')]z=0$$

$$(D+D')[D+D'-2]z=0$$

$$[D-(-D')+0][D-(-D')-2]z=0$$

$$m_1=-1;\quad m_2=-1$$

$$c_1=0;\quad c_2=2$$

$$z=f(y-x)+e^{2x}(y-x)$$

3. Solve $(D^2-2DD'+D'^2-3D+3D'+2)z=0$

Solution:

$$[(D-D'-1)(D-D'-2)]z=0$$

$$m_1=1;\quad m_2=1$$

$$c_1=1;\quad c_2=2$$

$$z=e^x f(y+x)+e^{2x}(y+x).$$

4. Solve $(D+1)(D+D'-1)=5e^x$

Solution:

$$[(D-0D'-(-1))(D-(-1)D'-1)]z=0$$

$$m_1=0;\quad m_2=-1$$

$$c_1=-1;\quad c_2=1$$

$$z=e^{-x}f(y)+e^x f(y-x)$$

UNIT – 2
SOLUTION OF PARTIAL DIFFERENTIAL EQUATION

METHOD OF SEPARATION OF VARIABLES

1. **Solve by using method of separation of variables** $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

Solution:

Let $z = X(x).Y(y)$

$z = XY$ ---------- (*)

$$\frac{\partial z}{\partial x} = X'Y$$

$$\frac{\partial z}{\partial y} = XY'$$

$$\frac{\partial^2 z}{\partial x^2} = X''Y$$

Substitute in given equation

$$X''Y - 2X'Y + XY' = 0$$

$$(X'' - 2X')Y = -XY'$$

$$\frac{(X'' - 2X')}{X} = -\frac{Y'}{Y} = k$$

Case (1):

Consider $\frac{(X'' - 2X')}{X} = k$

$$(X'' - 2X') = kX$$

$$X'' - 2X' - kX = 0$$

The above equation is quadratic in X

Here a = 1, b = -2, c = -k

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{2 \pm \sqrt{4 + 4k}}{2}$$

$$X = \frac{2 \pm 2\sqrt{1+k}}{2}$$

$$X = 1 \pm \sqrt{1+k}$$

$\therefore$ The roots are $X = 1+\sqrt{1+k}, \quad 1-\sqrt{1+k}$

$$X = c_1 e^{\left(1+\sqrt{1+k}\right)x} + c_2 e^{\left(1-\sqrt{1+k}\right)x} \text{ ---------- } (1)$$

Case (2):

Consider $\dfrac{-Y'}{Y} = k$

$$Y' = -kY$$

$$Y' + kY = 0$$

$$(D+k)Y = 0$$

$$m + k = 0$$

$$m = -k$$

$$Y = c_3 e^{-ky} \text{ ---------- } (2)$$

Substitute eqn (1) and (2) in (*)

$$z(x,y) = \left[c_1 e^{\left(1+\sqrt{1+k}\right)x} + c_2 e^{\left(1-\sqrt{1+k}\right)x}\right]\left[c_3 e^{-ky}\right]$$

$$z(x,y) = \left[c_1 c_3 e^{\left(1+\sqrt{1+k}\right)x} + c_2 c_3 e^{\left(1-\sqrt{1+k}\right)x}\right] e^{-ky}$$

$$z(x,y) = \left[A e^{\left(1+\sqrt{1+k}\right)x} + B e^{\left(1-\sqrt{1+k}\right)x}\right] e^{-ky}$$

2. Solve by using method of separation of variables $3\dfrac{\partial u}{\partial x} + 2\dfrac{\partial u}{\partial y} = 0$ **where** $u(x,0) = 4e^{-x}$

Solution:

Let $u = XY$ ---------- (*)

$$\frac{\partial u}{\partial x} = X'Y$$

$$\frac{\partial u}{\partial y} = XY'$$

Substitute in given equation

$$3X'Y + 2XY' = 0$$

$$3X'Y = -2XY'$$

$$\frac{X'}{X} = -\frac{2Y'}{3Y} = k$$

Case (1):

Consider $\frac{X'}{X} = k$

$$X' = kX$$

$$X' - kX = 0$$

$$(D-k)X = 0$$

$$m = k$$

$$X = c_1 e^{kx} \text{ ---------- } (1)$$

Case (2):

Consider $\frac{-2Y'}{3Y} = k$

$$Y' = -\tfrac{3}{2}kY$$

$$Y' + \tfrac{3}{2}kY = 0$$

$$\left(D + \tfrac{3}{2}k\right)Y = 0$$

$$m + \tfrac{3}{2}k = 0$$

$$m = -\tfrac{3}{2}k$$

$$Y = c_2 e^{-\frac{3}{2}ky} \text{ ---------- } (2)$$

Substitute eqn (1) and (2) in (*)

$$u(x,y) = c_1 e^{kx} c_2 e^{-\frac{3}{2}ky}$$

$$u(x,y) = Ae^{k\left(x - \frac{3}{2}y\right)} \text{ ----------(3)}$$

Given $u(x,0) = 4e^{-x}$

$$u(x,0) = Ae^{k(x)}$$

$$4e^{-x} = Ae^{k(x)}$$

Comparing both sides, we get

$$A = 4, \quad k = -1$$

Substitute in eqn (3)

$$u(x,y) = 4e^{-\left(x-\frac{3}{2}y\right)}$$

Classification of p.d.e

The second order pde is of the form

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0 \text{------} \rightarrow (1)$$

Equation (1) is classified as elliptic, parabolic and hyperbolic depending on

i) $B^2 - 4AC < 0,\ \ then\ \ it\ \ is\ \ elliptic\ equation$

ii) $B^2 - 4AC = 0,\ \ then\ \ it\ \ is\ \ parabolic\ equation$

iii) $B^2 - 4AC > 0,\ \ then\ \ it\ \ is\ hyperbolic\ equation$

3. **Find the nature of the partial differential equations** $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$

Solutions:

The given second order p.d.e is of the form $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$

Here A = 4, B = 4, C = 1

$B^2 - 4AC = 16 - 16 = 0$

$B^2 - 4AC = 0$

$\therefore$ **The given equation is parabolic**

4. **Classify** $x\, f_{xx} + y\, f_{yy} = 0, \quad x > 0, \quad y > 0$

Solution:

The given second order p.d.e is of the form $x\, f_{xx} + y\, f_{yy} = 0$

Here A = x, B = 0, C = y

$B^2 - 4AC = 0 - 4xy = -4xy$

But given $x > 0 \quad and \quad y > 0$

$B^2 - 4AC < 0$

$\therefore$ **The given equation is elliptic.**

5. **Classify** $u_{xx} - y^4 u_{yy} = 2y^3 u_y$

Solution:

The given second order p.d.e is of the form $u_{xx} - y^4 u_{yy} - 2y^3 u_y = 0$

Here $A = 1, \quad B = 0, \quad C = -y^4$

$B^2 - 4AC \quad = 4y^4$

If y = 0, we get $B^2 - 4AC = 0$

$\therefore$ **The given equation becomes parabolic.**

If $y > 0 \quad or \quad y < 0,$ **we get** $B^2 - 4AC > 0$

$\therefore$ **The given equation is hyperbolic.**

6. **Classify** $x^2 f_{xx} + \left(1 - y^2\right) f_{yy} = 0 \quad for \ -1 < y < 1, \ -\infty < x < \infty$

Solution:

The given second order p.d.e is of the form

$$x^2 f_{xx} + \left(1 - y^2\right) f_{yy} = 0$$

Here $A = x^2, \quad B = 0, \quad C = 1 - y^2$

$B^2 - 4AC \quad = -4x^2\left(1 - y^2\right)$

$B^2 - 4AC \quad = 4x^2\left(y^2 - 1\right)$

If x^2 **is always positive** $-\infty < x < \infty$

In $-1 < y < 1, \quad \left(y^2 - 1\right)$ **is negative**

$\therefore B^2 - 4AC < 0$

$\therefore$ **The given equation is elleptic.**

Derive one dimensional wave equations

The one dimensional wave equation is $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$ -------(1)

Let $\quad y = XT$ ---------- (2)

$$\frac{\partial y}{\partial x} = X'T$$

$$\frac{\partial y}{\partial t} = X\,T'$$

$$\frac{\partial^2 y}{\partial x^2} = X''T$$

$$\frac{\partial^2 y}{\partial t^2} = X\,T''$$

Substitute in equation (1)

$$XT'' = a^2 X'T$$

$$\frac{X''}{X} = \frac{T''}{a^2 T} = k$$

Equating

Consider $\frac{X''}{X} = k, \qquad \frac{T''}{a^2 T} = k$

$$X'' = kX, \quad and \quad T'' = ka^2 T$$

$$X'' - kX = 0 \quad -----(3)$$

$$T'' - ka^2 T = 0 ----(4)$$

The solution of equation (3) and (4) depends up on k

Case (1):

Let k be positive

i.e k = p^2

$$(3) \Rightarrow \quad X'' - p^2 X = 0$$

The axillary equation is

$$m^2 - p^2 = 0$$

$$m^2 = p^2$$

$$m = \pm p$$

$\therefore$ The roots are $X = A_1 e^{px} + A_2 e^{-px} \quad -----(5)$

$$(4) \Rightarrow \quad T'' - a^2 p^2 T = 0$$

The axillary equation is

$$m^2 - a^2 p^2 = 0$$

$$m^2 = a^2 p^2$$

$$m = \pm a\,p$$

$\therefore$ The roots are $T = A_3 e^{pat} + A_4 e^{-pat} \quad -----(6)$

Substitute equation (5) and (6) in (2)

$$y(x,t) = \left[A_1 e^{px} + A_2 e^{-px}\right]\left[A_3 e^{pat} + A_4 e^{-pat}\right] -----(I)$$

Case (2):

Let k be negative

i.e k = - p^2

$$(3) \Rightarrow \quad X'' + p^2 X = 0$$

The axillary equation is

$$m^2 + p^2 = 0$$

$$m^2 = -p^2$$

$$m = \pm pi$$

$\therefore$ The roots are $X = A_5 \cos px + A_6 \sin px \quad -----(7)$

$$(4) \Rightarrow \quad T'' + a^2 p^2 T = 0$$

The axillary equation is

$$m^2 + a^2 p^2 = 0$$

$$m^2 = -a^2 p^2$$

$$m = \pm a\,pi$$

$\therefore$ The roots are $T = A_7 \cos pat + A_8 \sin pat \quad -----(8)$

Substitute equation (7) and (8) in (2)

$$y(x,t) = \left[A_5 \cos px + A_6 \sin px\right]\left[A_7 \cos pat + A_8 \sin pat\right] -----(II)$$

Case (3):

Let k be zero

i.e k = 0

$$(3) \Rightarrow \quad X'' = 0$$

The axillary equation is

$$m^2 = 0$$

$$m = 0, 0$$

$\therefore$ The roots are $X = A_9 + A_{10} x \quad -----(9)$

$$(4) \Rightarrow \quad T'' = 0$$

The axillary equation is

$$m^2 = 0$$

$$m = 0, 0$$

$\therefore$ The roots are $T = A_{11} + A_{12}t \quad -----(10)$

Substitute equation (9) and (10) in (2)

$$y(x,t) = \left[A_9 + A_{10}x\right]\left[A_{11} + A_{12}t\right] -----(III)$$

Thus the various possible solutions are

1. $y(x,t) = \left[A_1 e^{px} + A_2 e^{-px}\right]\left[A_3 e^{pat} + A_4 e^{-pat}\right]$
2. $y(x,t) = \left[A_5 \cos px + A_6 \sin px\right]\left[A_7 \cos pat + A_8 \sin pat\right]$
3. $y(x,t) = \left[A_9 + A_{10}x\right]\left[A_{11} + A_{12}t\right]$

1. A string is stretched and fastened to two points x =0 and $x = l$ apart. Motion is started by displacing the string into the form $y = k\,(l\,x - x^2)$ from which it is released at time t =0 . Find the displacement of any point on the string at a distance of x from one end at time t .

Solution: The one dimensional wave equation is $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$

From the given problem, we get the following boundary conditions

(i) $y(0,t) = 0$ for all $t > 0$

(ii) $y(l,t) = 0$ for all $t > 0$

(iii) $\dfrac{\partial y}{\partial t}(x,0) = 0$ (initial velocity is zero), $0 < x < l$

(iv) $y\,(x,0) = k\,(l\,x - x^2)$, $0 < x < l$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \ldots\ldots..(1)$$

Applying the condition (i) **in (1) we get**

$y(0,t) = A(C\cos pat + D\sin pat) = 0$

i.e.., $(C\cos pat + D\sin pat) \neq 0$

$\therefore A = 0$

Substitute $A = 0$ in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \ldots\ldots(2)$$

Appling condition (ii) **in (2) we get**

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$$\sin pl = 0$$

$$pl = \sin^{-1}(0)$$

$$i.e..\quad pl = n\pi \qquad (\sin n\pi = 0)$$

$$i.e\ \ p = \frac{n\pi}{l}\ (\text{ n being an integer})$$

Now substituting $p = \dfrac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left[C\cos\left(\frac{n\pi at}{l}\right) + D\sin\left(\frac{n\pi at}{l}\right)\right] \ldots\ldots\ldots\ldots\ldots(3)$$

Before applyingcondition (iii), differentiating (3) partially w.r.t't'

$$\frac{\partial y}{\partial t}(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left(-C\sin\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) + D\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right)\right) \text{------- (4)}$$

Applying condition (iii) in eqn (4), we get

$$\frac{\partial y}{\partial t}(x,0) = B\sin\left(\frac{n\pi x}{l}\right)\left(D\frac{n\pi a}{l}\right) = 0$$

Here $\mathrm{B}\sin\dfrac{n\pi x}{l} \neq 0$ And $\dfrac{n\pi a}{l} \neq 0$

$$\therefore\quad D = 0$$

Therefore sub, D = 0 in (3) we get,

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)C\cos\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right) \text{---------- (5)}$$

Where $B_n = BC$

$\therefore$ The most general solutions of (5) can be written as

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}} \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

Applying the boundary conditions (iv) in (6) we get

$$y(x,0)=\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = k\,(l\,x - x^2)$$

To find B_n expand $k\,(l\,x - x^2)$ in a half range fouriersine series in the interval $(0,l)$

$$\therefore\ B_n = \frac{2}{l}\int_0^l k(lx-x^2)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2k}{l}\left[(lx-x^2)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)-(l-2x)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)+(-2)\left(\frac{\cos\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right)\right]_0^{\ell}$$

$$=\frac{2k}{l}\left[0-2\left(\frac{\cos n\pi}{\frac{n^3\pi^3}{l^3}}\right)-0+2\left(\frac{\cos 0}{\frac{n^3\pi^3}{l^3}}\right)\right] \qquad [\therefore\ \sin n\pi = 0]$$

$$= \frac{2k}{\ell}\times\frac{2\ell^3}{n^3\pi^3}\left[-(-1)^n+1\right]$$

$$B_n = \frac{4kl^2}{n^3\pi^3}\left[1-(-1)^n\right]$$

$$B_n=\begin{cases}\dfrac{8kl^2}{n^3\pi^3}, & n\ is\ odd\\ 0, & n\ is\ even\end{cases}$$

Substituting B_n in (6) we get

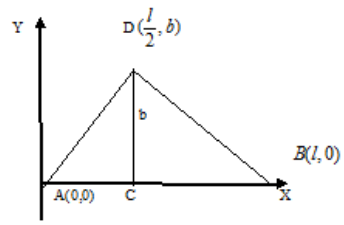
$$y(x,t) = \sum_{1,3,5..}^{\infty}\frac{8kl^2}{n^3\pi^3}\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)$$

2. A string is tightly stretched and its ends are fastened at two points $x=0$ and $x=l$. The midpoint of the string is displaced transversely through a small distance 'b' and the

string is released from the rest in that position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.

Solution:

First we find the equation of the string in its initial position ADB (see fig..)



The equation of the string (or line)AD is(0.0) $\left(\frac{l}{2}, b\right)$

Using $\left(\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}\right)$

$$\frac{y-0}{b-0}=\frac{x-0}{\frac{l}{2}-0} \Rightarrow \frac{y}{b}=\frac{2x}{l}$$

$$\therefore \boxed{y=\frac{2bx}{l}}, \quad 0<x<\frac{l}{2}$$

$\therefore$ The equation of the string in the interval $\left(0, \frac{l}{2}\right)$ is $\frac{2bx}{l}$

The equation of the string DB is $\left(\frac{l}{2}, b\right)$ and $(l, 0)$

$$\frac{y-b}{0-b}=\frac{x-\frac{l}{2}}{l-\frac{l}{2}} \Rightarrow \frac{y-b}{-b}=\frac{x-\frac{l}{2}}{\frac{l}{2}}$$

$$i.e.., \frac{y-b}{-b}=\frac{\frac{2x-l}{2}}{\frac{l}{2}} \Rightarrow \frac{y-b}{-b}=\frac{2x-l}{l}$$

$$i.e.., ly-lb=bl-2bx$$

$$ly=bl+lb-2bx \Rightarrow ly=2bl-2bx$$

$$ly=2b(l-x) \Rightarrow \boxed{y=\frac{2b}{l}(l-x)}, \quad \frac{l}{2}<x<l$$

$\therefore$ The equation of the string in the interval $\left(\frac{l}{2}, l\right)$ is $y = \frac{2b}{l}(l-x)$

Hence initially the displacement of the string is in the form

$$y(x,0) = \begin{cases} \frac{2bx}{l}, 0 < x < \frac{l}{2} \\ \frac{2b}{l}(l-x), \quad \frac{l}{2} < x < l \end{cases}$$

Now the wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

(i) $y(x,t) = 0$ for all $t > 0$

(ii) $y(l,t) = 0$ for all $t > 0$

(iii)) $\frac{\partial y(x,0)}{\partial t} = 0$ for every x in $(0,l)$

$$(iv)\ y(x,0) = \begin{cases} \frac{2bx}{l}, 0 < x < \frac{l}{2} \\ \frac{2b}{l}(l-x), \quad \frac{l}{2} < x < l \end{cases}$$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \ldots\ldots..(1)$$

Applying the condition (i) **in (1) we get**

$$y(0,t) = A(C\cos pat + D\sin pat) = 0$$

i.e.., $(C\cos pat + D\sin pat) \neq 0$

$\therefore A = 0$

Substitute $A = 0$ in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \ldots\ldots(2)$$

Appling condition (ii) **in (2) we get**

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$\sin pl = 0$

$pl = \sin^{-1}(0)$

$i.e..\quad pl = n\pi \quad (\sin n\pi = 0)$

$i.e\ \ p = \frac{n\pi}{l}$ (n being an integer)

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left[C\cos\left(\frac{n\pi at}{l}\right) + D\sin\left(\frac{n\pi at}{l}\right)\right] \ldots\ldots\ldots\ldots\ldots(3)$$

Before applying condition (iii), differentiating (3) partially w.r.t't'

$$\frac{\partial y}{\partial t}(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left(-C\sin\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) + D\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right)\right) \text{------- (4)}$$

Applying condition (iii) in eqn (4), we get

$$\frac{\partial y}{\partial t}(x,0) = B\sin\left(\frac{n\pi x}{l}\right)\left(D\frac{n\pi a}{l}\right) = 0$$

Here $B\sin\frac{n\pi x}{l} \neq 0$ and $\frac{n\pi a}{l} \neq 0$

$\therefore\ \ D = 0$

Therefore sub, D = 0 in (3) we get,

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)C\cos\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right) \text{---------- (5)}$$

Where $B_n = BC$

$\therefore$ The most general solutions of (5) can be written as

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Applying condition in (iv) **we get**

$$y(x,0) = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{l} = f(x) \quad \text{(say)}$$

Where $f(x) = \begin{cases} \frac{2bx}{l}, 0 < x < \frac{l}{2} \\ \frac{2b}{l}(l-x), \quad \frac{l}{2} < x < l \end{cases}$

To Find ' B_n '**expand the function in** $f(x)$ in a half range Fourier sine series in the interval $0 < x < l$

Therefore $B_n = \frac{2}{l}\int_0^{\ell} f(x)\sin\frac{n\pi x}{l}$

$$= \frac{2}{l}\left(\int_0^{l/2}\frac{2bx}{l}\sin\frac{n\pi x}{l}\,dx + \int_{l/2}^{l}\frac{2b}{l}(l-x)\sin\frac{n\pi x}{l}\,dx\right)$$

$$= \frac{4b}{l^2}\left[x\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)-\left(-\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^{l/2} + \left[(l-x)\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)+\left(-\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_{l/2}^{l}$$

$$= \frac{4b}{l^2}\left[\frac{-l^2}{2n\pi}\cos\frac{n\pi}{2}+\frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{2}+\frac{l^2}{2n\pi}\cos\frac{n\pi}{2}+\frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$= \frac{4b}{l^2}\left[\frac{2l^2}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$B_n = \frac{8b}{n^2\pi^2}\sin\frac{n\pi}{2}$$

Substituting (6) in the most general solution we get

$$y(x,t) = \sum_{n=1}^{\infty}\frac{8b}{n^2\pi^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}$$

3. A tightly stretched string with fixed points $x = 0$ **and** $x = l$ **is initially in a position given by** $y(x,0) = y_0\sin^3\frac{\pi x}{l}$ **. If it is released from the rest from this position find the displacement** y **at any distance** x **from the one end at any time** t **.**

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$

From the given problem, we get the following boundary conditions

(i) $y(0,t) = 0$ for all $t > 0$

(ii) $y(l,t) = 0$ for all $t > 0$

(iii) $\frac{\partial y(x,0)}{\partial t} = 0$, for all x (Initial velocity is zero)

$$(iv)\ y(x,0) = y_0 \sin^3 \frac{\pi x}{l}$$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \ldots\ldots..(1)$$

Applying the condition (i) in (1) we get

$$y(0,t) = A(C\cos pat + D\sin pat) = 0$$

But $(C\cos pat + D\sin pat) \neq 0$

$$\therefore A = 0$$

Substitute $A = 0$ in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \ldots\ldots(2)$$

Appling condition (ii) in (2) we get

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$$\sin pl = 0$$

$$pl = \sin^{-1}(0)$$

$$i.e..\quad pl = n\pi \qquad (\sin n\pi = 0)$$

$$i.e\ \ p = \frac{n\pi}{l} \text{ (n being an integer)}$$

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left[C\cos\left(\frac{n\pi at}{l}\right) + D\sin\left(\frac{n\pi at}{l}\right)\right] \ldots\ldots\ldots\ldots\ldots(3)$$

Before applying condition (iii), differentiating (3) partially w.r.t't'

$$\frac{\partial y}{\partial t}(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left(-C\sin\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) + D\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right)\right) \text{-------- (4)}$$

Applying condition (iii) in eqn (4), we get

$$\frac{\partial y}{\partial t}(x,0) = B\sin\left(\frac{n\pi x}{l}\right)\left(D\frac{n\pi a}{l}\right) = 0$$

Here $\mathrm{B}\sin\frac{n\pi x}{l} \neq 0$ and $\frac{n\pi a}{l} \neq 0$

$$\therefore\ \ D = 0$$

Therefore sub, D = 0 in (3) we get,

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)C\cos\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right) \text{---------- (5)}$$

Where $B_n = BC$

$\therefore$ The most general solutions of (5) can be written as

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Applying condition (iv) in (6) we get,

$$y(x,0) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{l}\right) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$$

We know that $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

$$\therefore \sin^3\frac{\pi x}{l} = \frac{1}{4}\left(3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right)$$

Substitute in above equations

$$\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = \frac{y_0}{4}\left(3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right)$$

$$i.e., B_1 \sin\frac{\pi x}{l} + B_2 \sin\frac{2\pi x}{l} + B_3 \sin\frac{3\pi x}{l} + \ldots\ldots\ldots\ldots = \frac{y_0}{4}\left(3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right)$$

By equating the coefficient on either side, we get

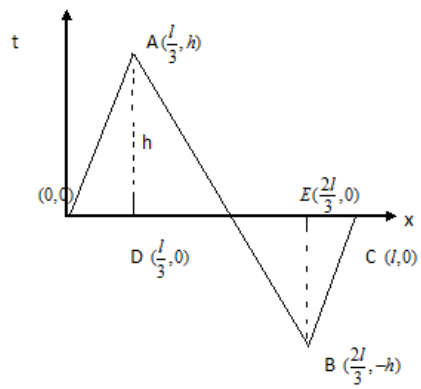
$$B_1 = \frac{3y_0}{4}, B_3 = \frac{-y_0}{4}, B_2 = 0, B_3 = 0, B_4 = 0.....$$

Substituting these values of $c_1, c_2, c_3....$ in (1)

$$y(x,t) = \frac{3y_0}{4}\sin\frac{\pi x}{l}\cos\frac{\pi at}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l}\cos\frac{3\pi at}{l}$$

4. The points of trisection of a string are pulled aside through a distance 'h' on opposite sides of position of equilibrium and the string is released from rest. Find the displacement of the string at any subsequent time.

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$



W.K.T The equ. of line joining points (x_1, y_1) & (x_2, y_2) is given by

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

To find Equ of OA:O(0,0) &A $\left(\frac{l}{3}, h\right)$

$$\frac{y-0}{h-0} = \frac{x-0}{\frac{l}{3}-0}$$

$$\frac{y}{h} = \frac{3x}{l}$$

$$y = \frac{3hx}{l}, \quad 0 < x < \frac{l}{3}$$

Equation of AB; $A\left(\frac{l}{3}, h\right)$ & $B\left(\frac{2l}{3}, -h\right)$

$$\frac{y-h}{-h-h} = \frac{x-\frac{l}{3}}{\frac{2l}{3}-\frac{l}{3}}$$

$$\frac{y-h}{-2h} = \frac{(3x-l)/3}{\frac{l}{3}}$$

$$y-h = -2h\frac{(3x-l)}{l}$$

$$y = h-2h\frac{(3x-l)}{l}$$

$$y = \frac{hl-6hx+2hl}{l}$$

$$y=\frac{3hl-6hx}{l}\ ,\frac{l}{3}<x<\frac{2l}{3}$$

$$y=\frac{3h(l-2x)}{l}\ ,\frac{l}{3}<x<\frac{2l}{3}$$

Equation of BC; $B\left(\frac{2l}{3},-h\right)$ & $C(l,0)$

$$\frac{y+h}{0+h}=\frac{x-\frac{2l}{3}}{l-\frac{2l}{3}}\Rightarrow\frac{y+h}{h}=\frac{(3x-2l)/3}{(3l-2l)/3}$$

$$y+h=\frac{h(3x-2l)}{3l}$$

$$y=-h+\frac{h(3x-2l)}{l}$$

$$y=\frac{-hl+3hx-2hl}{l}$$

$$y=\frac{-3hl+3hx}{l}$$

$$y=\frac{3h(x-l)}{l}\quad,\ \frac{2l}{3}<x<l$$

$$\therefore\ f(x)=\begin{cases}\frac{3hx}{l},0<x<\frac{l}{3}\\ \frac{3h(l-2x)}{l},\ \frac{l}{3}<x<\frac{2l}{3}\\ \frac{3h(x-l)}{l},\ \frac{2l}{3}<x<l\end{cases}$$

The given problem, we get the following boundary conditions

$(i)\ y(0,t)=0$ for all $t>0$

$(ii)\ y(l,t)=0$ for all $t>0$

$(iii)\ \frac{\partial y(x,0)}{\partial t}=0$, for all x

$$(iv)\ y(x,0)==\begin{cases}\frac{3hx}{l},0<x<\frac{l}{3}\\ \frac{3h(l-2x)}{l},\ \frac{l}{3}<x<\frac{2l}{3}\\ \frac{3h(x-l)}{l},\ \frac{2l}{3}<x<l\end{cases}$$

The solution equation is

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \ldots\ldots..(1)$$

Applying the condition (i) **in (1) we get**

$$y(0,t) = A(C\cos pat + D\sin pat) = 0$$

But $(C\cos pat + D\sin pat) \neq 0$

$$\therefore A = 0$$

Substitute $A = 0$ in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \ldots\ldots(2)$$

Appling condition (ii) **in (2) we get**

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$$\sin pl = 0$$

$$pl = \sin^{-1}(0)$$

$$i.e.. \quad pl = n\pi \qquad (\sin n\pi = 0)$$

$$i.e \ p = \frac{n\pi}{l} \text{ (n being an integer)}$$

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left[C\cos\left(\frac{n\pi at}{l}\right) + D\sin\left(\frac{n\pi at}{l}\right)\right] \ldots\ldots\ldots\ldots\ldots(3)$$

Before applying condition (iii), differentiating (3) partially w.r.t‘t’

$$\frac{\partial y}{\partial t}(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left(-C\sin\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) + D\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right)\right) \text{-------} (4)$$

Applying condition (iii) in eqn (4), we get

$$\frac{\partial y}{\partial t}(x,0) = B\sin\left(\frac{n\pi x}{l}\right)\left(D\frac{n\pi a}{l}\right) = 0$$

Here $B\sin\frac{n\pi x}{l} \neq 0$ and $\frac{n\pi a}{l} \neq 0$

$$\therefore \quad D = 0$$

Therefore sub, D = 0 in (3) we get,

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)C\cos\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right) \text{---------- (5)}$$

Where $B_n = BC$

$\therefore$ The most general solutions of (5) can be written as

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}} \quad \ldots\ldots\ldots\ldots\ldots \ (6)$$

Applying condition (iv) in (6) we get,

$$y(x,0) = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{l} = f(x)$$

$$B_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}$$

$$B_n = \frac{2}{l}\left[\int_0^{\frac{l}{3}} \frac{3hx}{l}\sin\frac{n\pi x}{l}dx + \int_{\frac{l}{3}}^{\frac{2l}{3}} \frac{3h(l-2x)}{l}\sin\frac{n\pi x}{l}dx + \int_{\frac{2l}{3}}^{l} \frac{3h(x-l)}{l}\sin\frac{n\pi x}{l}dx\right]$$

$$B_n = \frac{6h}{l^2}\left[\int_0^{\frac{l}{3}} x\sin\frac{n\pi x}{l}dx + \int_{\frac{l}{3}}^{\frac{2l}{3}} (l-2x)\sin\frac{n\pi x}{l}dx + \int_{\frac{2l}{3}}^{l} (x-l)\sin\frac{n\pi x}{l}dx\right]$$

$$B_n = \frac{6h}{l^2}\left[x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (1)\left(\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^{\frac{l}{3}} + \left[(l-2x)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (-2)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_{\frac{l}{3}}^{\frac{2l}{3}} + \left[(x-l)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_{\frac{2l}{3}}^{l}$$

$$B_n = \frac{6h}{l^2}\left[\frac{-l^2}{3n\pi}\cos\frac{n\pi}{3} + \frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{3}\right] + \left[\frac{l^2}{3n\pi}\cos\frac{2n\pi}{3} - \frac{2l^2}{n^2\pi^2}\sin\frac{2n\pi}{3}\right]$$

$$-\left[\frac{-l^2}{3n\pi}\cos\frac{n\pi}{3} - \frac{2l^2}{n^2\pi^2}\sin\frac{n\pi}{3}\right] + \left[(0-0) - (\frac{l^2}{3n\pi}\cos\frac{2n\pi}{3} + \frac{l^2}{n^2\pi^2}\sin\frac{2n\pi}{3}\right]$$

$$B_n = \frac{6h}{l^2}\left[\frac{-l^2}{3n\pi}\cos\frac{n\pi}{3} + \frac{l^2}{n^2\pi^2}\sin\frac{n\pi}{3} + \frac{l^2}{3n\pi}\cos\frac{2n\pi}{3} - \frac{2l^2}{n^2\pi^2}\sin\frac{2n\pi}{3}\right.$$

$$\left. + \frac{l^2}{3n\pi}\cos\frac{n\pi}{3} + \frac{2l^2}{n^2\pi^2}\sin\frac{n\pi}{3} - \frac{l^2}{3n\pi}\cos\frac{2n\pi}{3} - \frac{l^2}{n^2\pi^2}\sin\frac{2n\pi}{3}\right]$$

$$B_n = \frac{6h}{l^2}\left[\frac{3l^2}{n^2\pi^2}\sin\frac{n\pi}{3} - \frac{3l^2}{n^2\pi^2}\sin\frac{2n\pi}{3}\right]$$

$$B_n = \frac{18hl^2}{l^2n^2\pi^2}\left[\sin\frac{n\pi}{3} - \sin\frac{2n\pi}{3}\right]$$

$$B_n = \frac{18h}{n^2\pi^2}\left[\sin\frac{n\pi}{3} - \sin\frac{2n\pi}{3}\right]$$

$$B_n = \frac{18h}{n^2\pi^2}\left[\sin\frac{n\pi}{3} - \sin(n\pi - \frac{n\pi}{3})\right]$$

$$B_n = \frac{18h}{n^2\pi^2}\left[\sin\frac{n\pi}{3} + (-1)^n\sin\frac{n\pi}{3}\right]$$

$$B_n = \frac{18h}{n^2\pi^2}\sin\frac{n\pi}{3}\left[1 + (-1)^n\right]$$

$$B_n = \begin{cases} \frac{36h}{n^2\pi^2}\sin\frac{n\pi}{3}, n = even \\ 0, \qquad n = odd \end{cases}$$

$$\therefore\ y(x,t) = \sum_{n=2,4,\ldots}^{\infty}\frac{36h}{n^2\pi^2}\sin\frac{n\pi}{3}\cos\frac{n\pi at}{l}\sin\frac{n\pi x}{l}$$

Non – Zero Velocity Problems

1. **If a string of length 'l' is initially at rest in its equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0\sin^3\frac{\pi x}{l}, 0 < x < l$. Determine the displacement function $y(x,t)$.**

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

(i) $y(0,t) = 0$ for all $t > 0$

(ii) $y(l,t) = 0$ for all $t > 0$

(iii) $y(x,0) = 0$, for all x in $(0,l)$ since the string has no displacement in its equilibrium Position.

(iv) $\frac{\partial y(x,0)}{\partial t} = v_0\sin^3\frac{\pi x}{l}$, for all x in $(0,l)$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \text{…….(1)}$$

Applying the condition (i) **in (1) we get**

$$y(0,t) = A(C\cos pat + D\sin pat) = 0$$

But $(C\cos pat + D\sin pat) \neq 0$

$$\therefore A = 0$$

Substituting in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \text{……..(2)}$$

Now applying condition (ii) **in (2) we get**

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$$\sin pl = 0$$

$$pl = \sin^{-1}(0)$$

$$i.e.. \quad pl = n\pi \qquad (\sin n\pi = 0)$$

$$i.e \;\; p = \frac{n\pi}{l} \text{ (n being an integer)}$$

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\frac{n\pi x}{l}\left[C\cos\frac{n\pi at}{l} + D\sin\frac{n\pi at}{l}\right] \qquad \text{…….(3)}$$

Applying condition (iii) in (3) we get

$$y(x,0) = B\sin\frac{n\pi x}{l}.C = 0$$

But $B \neq 0 \quad and \quad \sin\frac{n\pi x}{l} \neq 0$

$$\therefore C = 0$$

Substituting in (3) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)D\sin\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right) \text{---------- (4)where } B_n = BD$$

The most general equation of equation (4) is

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}} \qquad \text{……(5)}$$

Before applying condition (iv), find $\frac{\partial y(x,t)}{\partial t}$ differentiating (5) partially w.r.t't' we get

$$\frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) \text{----------(6)}$$

Putting $t = 0$ in (6) we get

$$\frac{\partial y(x,0)}{\partial t} = \sum_{n=1}^{\infty} B_n \left(\frac{n\pi a}{l}\right)\sin\frac{n\pi x}{l} = v_0 \sin^3\frac{\pi x}{l} \quad \text{[By condition (iv)]}$$

But $\sin^3\frac{\pi x}{l} = \frac{1}{4}\left(3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right)$

$$i.e.. \sum_{n=1}^{\infty} B_n \left(\frac{n\pi a}{l}\right)\sin\frac{n\pi x}{l} = \frac{v_0}{4}\left(3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right) \text{……(7)}$$

From (7) we get

$$B_1\frac{\pi a}{l}\sin\frac{\pi x}{l} + B_2\frac{2\pi a}{l}\sin\frac{2\pi x}{l} + B_3\frac{3\pi a}{l}\sin\frac{3\pi x}{l} + ... = \frac{3v_0}{4}\sin\frac{\pi x}{l} - \frac{v_0}{4}\sin\frac{3\pi x}{l}$$

By equating the co-efficient we get

$$B_1\frac{\pi a}{l} = \frac{3v_0}{4},\ B_2 = 0,\quad B_3\frac{3\pi a}{l} = -\frac{v_0}{4},\quad B_4 = 0,\ B_5 = 0,.....$$

$\therefore B_1 = \frac{3v_0}{4}\times\frac{l}{\pi a}$ and $B_3 = -\frac{v_0 l}{12\pi a}$ the remaining B_n 's are zero.

Substituting these values of c's in (5) we get

$$y(x,t) = \frac{3v_0 l}{4\pi a}\sin\frac{\pi x}{l}\sin\frac{\pi at}{l} - \frac{v_0 l}{12\pi a}\sin\frac{3\pi x}{l}\sin\frac{3\pi at}{l}$$

7. **A tightly stretched stringwth fixed end points $x = 0$ *and* $x = l$ is initially at rest in its equilibrium position. If it is set vibrating string giving each point the velocity $\lambda x(\ell - x)$. Determine the displacement function $y(x,t)$.**

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

$(i)\ y(0,t)=0$ for all $t>0$

$(ii)\ y(l,t)=0$ for all $t>0$

$(iii)\ y(x,0)=0$, for all x in $(0,l)$ since the string has no displacement in its equilibrium Position.

$(iv)\ \dfrac{\partial y(x,0)}{\partial t}=\lambda x(\ell-x)$, for all x in $(0,l)$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t)=(A\cos px+B\sin px)(C\cos pat+D\sin pat) \qquad \text{…….(1)}$$

Applying the condition (i) in (1) we get

$$y(0,t)=A(C\cos pat+D\sin pat)=0$$

But $(C\cos pat+D\sin pat)\neq 0$

$$\therefore A=0$$

Substituting in (1) we get

$$y(x,t)=B\sin px(C\cos pat+D\sin pat) \qquad \text{……..(2)}$$

Now applying condition (ii) in (2) we get

$$y(\ell,t)=B\sin pl(C\cos pat+D\sin pat)=0$$

Here $C\cos pat+D\sin pat\neq 0$ and $B\neq 0$ (therefore it is defined for all t)

Therefore we take $\sin pl=0$

$$pl=\sin^{-1}(0)$$

$i.e..\ pl=n\pi \quad (\sin n\pi=0)$

$i.e\ p=\dfrac{n\pi}{l}$ (n being an integer)

Now substituting $p=\dfrac{n\pi}{l}$ in (2) we get

$$y(x,t)=B\sin\frac{n\pi x}{l}\left[C\cos\frac{n\pi at}{l}+D\sin\frac{n\pi at}{l}\right] \qquad \text{…….(3)}$$

Applying condition (iii) in (3) we get

$$y(x,0)=B\sin\frac{n\pi x}{l}.C=0$$

But $B \neq 0$ *and* $\sin\frac{n\pi x}{l} \neq 0$

$\therefore C = 0$

Substituting in (3) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right) D\sin\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right) \text{ ---------- (4)} \qquad \text{where } B_n = BD$$

The most general equation of equation (4) is

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}} \qquad \text{.....(5)}$$

Before applying condition (iv), find $\frac{\partial y(x,t)}{\partial t}$ differentiating (5) partially w.r.t 't' we get

$$\frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) \text{----------(6)}$$

Putting $t = 0$ in (6) we get

$$\frac{\partial y(x,0)}{\partial t} = \sum_{n=1}^{\infty} C_n \left(\frac{n\pi a}{l}\right)\sin\frac{n\pi x}{l} = \lambda x(\ell - x), \text{ where } C_n = B_n\left(\frac{n\pi a}{\ell}\right)$$

Now to find 'B_n'expand the function in $f(x)$ in a half range Fourier sine series in the interval $0 < x < l$

Where $C_n = \frac{2}{l}\int_0^{\ell} f(x)\sin\frac{n\pi x}{l}\,dx$

$$C_n = \frac{2}{l}\int_0^{\ell} \lambda\left(\ell x - x^2\right)\sin\frac{n\pi x}{l}$$

$$= \frac{2\lambda}{\ell}\left[\left(\ell x - x^2\right)\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (\ell - 2x)\left(-\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + (-2)\left(\frac{\cos\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right)\right]_0^l$$

$$= \frac{2\lambda}{l}\left[-\left(\ell x - x^2\right)\left(\frac{l}{n\pi}\right)\cos\left(\frac{n\pi x}{\ell}\right) - \left(\frac{2l^3}{n^3\pi^3}\right)\cos\left(\frac{n\pi x}{\ell}\right)\right]_0^{\ell}$$

$$=\frac{-2\lambda}{l}\left\{\frac{2l^3}{n^3\pi^3}\left[(-1)^n-1\right]\right\}$$

$$C_n \quad =\begin{cases}\dfrac{8\lambda\ell^2}{n^3\pi^3}, & if\ \ n\ is\ \ odd\\ 0, & if\ n\ \ is\ \ even\end{cases}$$

But $\quad C_n = B_n\left(\dfrac{n\pi a}{\ell}\right)$

$$B_n = C_n\left(\frac{\ell}{n\pi a}\right)$$

$$B_n =\begin{cases}\dfrac{8\lambda\ell^2}{n^3\pi^3}\times\left(\dfrac{\ell}{n\pi a}\right), & if\ \ n\ is\ \ odd\\ 0, & if\ n\ \ is\ \ even\end{cases}$$

Substituting (6) in the most general solution we get

$$y(x,t)=\frac{8\lambda\ell^3}{a\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\sin\left(\frac{n\pi x}{\ell}\right)\sin\left(\frac{n\pi at}{l}\right)$$

2. **If a string of length l is initially at rest in its equilibrium position and each of its points isgiven a velocity v such that** $v=\begin{cases}cx & 0<x<l/2\\ c(l-x) & l/2<x<l\end{cases}$ **Find the displacement of the string at any time t.**

Solution: The one dimensional wave equation is $\dfrac{\partial^2 y}{\partial t^2}=a^2\dfrac{\partial^2 y}{\partial x^2}$

The boundary conditions are

(i) $y(0,t)=0$ for all $t>0$

(ii) $y(l,t)=0$ for all $t>0$

(iii) $y(x,0)=0$, for all x in $(0,l)$ since the string has no displacement in its equilibrium Position.

(iv) $\dfrac{\partial y(x,0)}{\partial t}=\begin{cases}cx & 0<x<l/2\\ c(l-x) & l/2<x<l\end{cases}$, for all x in $(0,l)$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t)=(A\cos px+B\sin px)(C\cos pat+D\sin pat) \qquad \text{…….(1)}$$

Applying the condition (i) **in (1) we get**

$$y(0,t)=A(C\cos pat+D\sin pat)=0$$

But $(C\cos pat + D\sin pat) \neq 0$

$$\therefore A = 0$$

Substituting in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \text{……..(2)}$$

Now applying condition (ii) **in (2) we get**

$$y(\ell,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take $\sin pl = 0$

$$pl = \sin^{-1}(0)$$

$i.e..$ $pl = n\pi$ $(\sin n\pi = 0)$

$i.e$ $p = \dfrac{n\pi}{l}$ (n being an integer)

Now substituting $p = \dfrac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\frac{n\pi x}{l}\left[C\cos\frac{n\pi at}{l} + D\sin\frac{n\pi at}{l}\right] \qquad \text{…….(3)}$$

Applying condition (iii) in (3) we get

$$y(x,0) = B\sin\frac{n\pi x}{l}.C = 0$$

But $B \neq 0$ *and* $\sin\dfrac{n\pi x}{l} \neq 0$

$$\therefore C = 0$$

Substituting in (3) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)D\sin\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n\sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right) \text{ ---------- (4)} \qquad \text{where } B_n = BD$$

The most general equation of equation (4) is

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}} \qquad \text{……(5)}$$

Before applying condition (iv), find $\frac{\partial y(x,t)}{\partial t}$ differentiating (5) partially w.r.t 't' we get

$$\frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) \text{----------(6)}$$

Putting $t = 0$ in (6) we get

$$\frac{\partial y(x,0)}{\partial t} = \sum_{n=1}^{\infty} C_n \left(\frac{n\pi a}{l}\right)\sin\frac{n\pi x}{l} = \begin{cases} cx & 0 < x < l/2 \\ c(l-x) & l/2 < x < l \end{cases}, \text{ where } C_n = B_n\left(\frac{n\pi a}{\ell}\right)$$

Now to find 'B_n'expand the function in $f(x)$ in a half range Fourier sine series in the interval $0 < x < l$

Where $C_n = \frac{2}{l}\int_0^{\ell} f(x)\sin\frac{n\pi x}{l}\,dx$

$$= \frac{2}{\ell}\left(\int_0^{l/2} cx\sin\left(\frac{n\pi x}{l}\right)dx + \int_{l/2}^{l} c(l-x)\sin\left(\frac{n\pi x}{l}\right)dx\right)$$

$$= \frac{2c}{\ell}\left[x\left(-\frac{\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right) - \left(-\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^{l/2} + \left[(l-x)\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) + \left(-\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]$$

$$= \frac{2c}{l}\left[\frac{-l^2}{2n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \frac{l^2}{2n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{2c}{l}\left[\frac{2l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$C_n = \frac{4c\ell}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

But $C_n = B_n\left(\frac{n\pi a}{\ell}\right)$

$$B_n = C_n\left(\frac{\ell}{n\pi a}\right)$$

$$B_n = \left\{\frac{4c\ell}{n^2\pi^2} \times \left(\frac{\ell}{n\pi a}\right)\sin\left(\frac{n\pi}{2}\right)\right.$$

$$B_n = \left\{\frac{4c\ell^2}{n^3\pi^3 a}\sin\left(\frac{n\pi}{2}\right)\right.$$

Substituting (6) in the most general solution we get

$$y(x,t)=\frac{4c\ell^2}{\pi^3 a}\sum_{n=1}^{\infty}\frac{1}{n^3}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{\ell}\right)\sin\left(\frac{n\pi at}{l}\right)$$

UNIT III

HEAT FLOW EQUATIONS

ONE DIMENSIONAL HEAT EQUATIONS:

1.Obtain the solution of one dimensional wave equation.

Solution:

Let $u = XT$ ---------- (*)

$$\frac{\partial u}{\partial x} = X'T$$

$$\frac{\partial^2 u}{\partial x^2} = X''T$$

$$\frac{\partial u}{\partial t} = X\,T'$$

Substitute in given equation

$$T'X = \alpha^2 X''T$$

$$\frac{X''}{X} = \frac{1}{\alpha^2}\frac{T'}{T} = k$$

Equating

$\frac{X''}{X} = k$	$\frac{1}{\alpha^2}\frac{T'}{T} = k$
$X'' = kX$	$T' = \alpha^2 kT$
$X'' - kX = 0 \;\; ----\rightarrow (3)$	$T' - \alpha^2 kT = 0 \;\; ----\rightarrow (4)$

Case (i)

Let k be positive

i.e $k = p^2$

$(3) \Rightarrow \quad X'' - p^2 X = 0$

The axillary equation is

$$m^2 - p^2 = 0$$

$$m^2 = p^2$$

$$m = \pm p$$

$\therefore$ The roots are $X = A_1 e^{px} + A_2 e^{-px} \quad -----(5)$

$$(4) \Rightarrow \quad T' - \alpha^2 p^2 T = 0$$

The auxiliary equation is

$$m - \alpha^2 p^2 = 0$$

$$m = \alpha^2 p^2$$

$\therefore$ The roots are $T = A_3 e^{\alpha^2 p^2 t} \quad -----(6)$

Substitute equation (5) and (6) in (2)

$$u(x,t) = \left[A_1 e^{px} + A_2 e^{-px} \right] A_3 e^{\alpha^2 p^2 t} \quad -----(I)$$

Case (2):

Let k be negative

i.e k = - p^2

$$(3) \Rightarrow \quad X'' + p^2 X = 0$$

The auxiliary equation is

$$m^2 + p^2 = 0$$

$$m^2 = -p^2$$

$$m = \pm pi$$

$\therefore$ The roots are $X = A_4 \cos px + A_5 \sin px \quad -----(7)$

$$(4) \Rightarrow \quad T' + \alpha^2 p^2 T = 0$$

The auxiliary equation is

$$m + \alpha^2 p^2 = 0$$

$$m = -\alpha^2 p^2$$

$\therefore$ The roots are $T = A_6 e^{-\alpha^2 p^2 t} \quad -----(8)$

Substitute equation (7) and (8) in (2)

$$u(x,t) = \left[A_4 \cos px + A_5 \sin px \right] A_6 e^{-\alpha^2 p^2 t} \quad -----(II)$$

Case (3):

Let k be zero

i.e k = 0

$$(3) \Rightarrow \quad X'' = 0$$

The auxiliary equation is

$$m^2 = 0$$

$$m = 0, 0$$

$\therefore$ The roots are $X = A_7 + A_8 x \quad -----(9)$

$$(4) \Rightarrow \quad T' = 0$$

The auxiliary equation is

$$m = 0$$

$\therefore$ The roots are $T = A_9 \quad -----(10)$

Substitute equation (9) and (10) in (2)

$$u(x,t) = \left[A_7 + A_8 x\right] A_9 \quad -----(III)$$

Thus the various possible solutions are

3. $u(x,t) = \left[A_1 e^{px} + A_2 e^{-px}\right] A_3 e^{\alpha^2 p^2 t}$

4. $u(x,t) = \left[A_4 \cos px + A_5 \sin px\right] A_6 e^{-\alpha^2 p^2 t}$

5. $u(x,t) = \left[A_7 + A_8 x\right] A_9$

2. **Find the solution of the equation** $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ **that satisfies the condition.**

$u(0,t) = 0, \quad u(l,t) = 0$ for $t \geq 0$ and $u(x,0) = u_0$ for all x **(May 2011)**

Solution:

The one dimensional heat equation is given by $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$,

The suitable solution is $u(x,t) = (A\cos px + B\sin px)e^{-\alpha^2 p^2 t}$ ---------(1)

The boundary conditions are

(i) $u(0,t) = 0$ for all $t > 0$

(ii) $u(l,t) = 0$ for all $t > 0$

(iii) $u(x,0) = u_0$ for x in $(0,l)$

Applying condition (i) in (1) we get

$$u(0,t) = Ae^{-\alpha^2 p^2 t} = 0$$

$$But \quad e^{-\alpha^2 p^2 t} \neq 0 \qquad \therefore A = 0$$

Substituting $A=0$ in (1) we get

$$u(x,t)=B\sin pxe^{-\alpha^2p^2t}=0 \qquad \text{----------(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t)=B\sin ple^{-\alpha^2p^2t}=0$$

Here $B\neq 0$ since if $B=0$ we get trivial solution.

$e^{-\alpha^2p^2t}\neq 0$ since it is true for all 't'

Hence $\sin pl=0$ i.e., $pl=n\pi$

Therefore $p=\dfrac{n\pi}{l}$

Substituting $p=\dfrac{n\pi}{l}$ in (2) we get

$$u(x,t)=B\sin\left(\frac{n\pi x}{l}\right)e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}} \qquad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l} = u_0$$

To find B_n expand x in $(0,l)$ in a half range Fourier sine series.

$$B_n=\frac{2}{l}\int_0^l u_0\sin\frac{n\pi x}{l}dx$$

$$=\frac{2u_0}{l}\left[\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right]_0^l=\frac{2u_0}{l}\left[\frac{-l\cos n\pi}{n\pi}+\frac{l}{n\pi}\right]$$

$$=\frac{2lu_0}{n\pi}\left[1-(-1)^n\right] \quad \left[\because \cos n\pi=(-1)^n\right] \qquad \text{----(4)}$$

Substituting (4) in (3) we get

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2lu_0}{n\pi}\left(1-(-1)^n\right)\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}$$

3 . **Solve the equation** $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, **subject to the boundary conditions**

$u(0,t) = 0, u(l,t) = 0, u(x,0) = x$ **(Dec 2014)**

Solution:

The one dimensional heat equation is given by $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$,

The suitable solution is

$$u(x,t) = (A\cos px + B\sin px)e^{-\alpha^2 p^2 t} \qquad \text{----(1)}$$

The boundary conditions are

(i) $u(0,t) = 0$ for all $t > 0$

(ii) $u(l,t) = 0$ for all $t > 0$

(iii) $u(x,0) = x$ for x in $(0,l)$

Applying condition (i) in (1) we get

$$u(0,t) = Ae^{-\alpha^2 p^2 t} = 0$$

$$But \;\; e^{-\alpha^2 p^2 t} \neq 0 \qquad \therefore A = 0$$

Substituting $A = 0$ in (1) we get

$$u(x,t) = B\sin pxe^{-\alpha^2 p^2 t} = 0 \qquad \text{----(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t) = B\sin ple^{-\alpha^2 p^2 t} = 0$$

Here $B \neq 0$ since if $B = 0$ we get trivial solution.

$e^{-\alpha^2 p^2 t} \neq 0$ since it is true for all 't'

Hence $\sin pl = 0$ i.e., $pl = n\pi$

Therefore $p = \frac{n\pi}{l}$

Substituting $p = \frac{n\pi}{l}$ in (2) we get

$$u(x,t) = B\sin \frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}} \quad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}= \quad x$$

To find B_n expand x in $(0,l)$ in a half range Fourier sine series.

$$B_n=\frac{2}{l}\int_0^l x\sin\frac{n\pi x}{l}dx$$

$$=\frac{2}{l}\left[x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)-1\left(\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l$$

$$=\frac{2}{l}\left[\frac{-l^2\cos n\pi}{n\pi}\right]$$

$$=\frac{2l}{n\pi}(-1)^{n+1}\left[\because \cos n\pi=(-1)^n\right] \quad \text{----(4)}$$

Substituting (4) in (3) we get

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2l}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}$$

4. A uniform bar of length 'l' thorough which heat flows is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given $K\left(lx-x^2\right)$,for $0<x<l$. Find the temperature distribution in the bar after time 't'. (May 2013)

Solution:

The one dimensional heat equation is given by $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$,

The suitable solution is

$$u(x,t)=(A\cos px+B\sin px)e^{-\alpha^2p^2t} \quad \text{----(1)}$$

The boundary conditions are

(i) $u(0,t)=0$ for all $t>0$

$(ii)\ u(l,t)=0$ for all $t>0$

$(iii)\ u(x,0)=K\left(lx-x^2\right)$ for x in $(0,l)$

Applying condition (i) in (1) we get

$$u(0,t)=Ae^{-\alpha^2p^2t}=0$$

$$But\ \ e^{-\alpha^2p^2t}\neq 0 \qquad \therefore A=0$$

Substituting $A=0$ in (1) we get

$$u(x,t)=B\sin pxe^{-\alpha^2p^2t}=0 \qquad \text{----(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t)=B\sin ple^{-\alpha^2p^2t}=0$$

Here $B\neq 0$ since if $B=0$ we get trivial solution.

$e^{-\alpha^2p^2t}\neq 0$ since it is true for all 't'

Hence $\sin pl=0$ i.e., $pl=n\pi$

Therefore $p=\dfrac{n\pi}{l}$

Substituting $p=\dfrac{n\pi}{l}$ in (2) we get

$$u(x,t)=B\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}} \qquad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l} \quad = \quad K\left(lx-x^2\right)$$

To find B_n expand x in $(0,l)$ in a half range Fourier sine series.

$$B_n=\frac{2}{l}\int_0^l K\left(lx-x^2\right)\sin\frac{n\pi x}{l}dx$$

$$=\frac{2K}{l}\left[\left(lx-x^2\right)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)-\left(l-2x\right)\left(\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right)+\left(-2\right)\left(\frac{\cos\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^l$$

$$=\frac{4Kl^2}{n^3\pi^3}\left(1-(-1)^n\right) \qquad \text{----(4)}$$

Substituting (6) in (3) we get

$$u(x,t)=\sum_{n=1}^{\infty}\frac{4Kl^2}{n^3\pi^3}\left(1-(-1)^n\right)\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}$$

5. An insulated rod of length '*l*' has its ends A and B kept at $0^\circ c$ and 100^0C respectively, until steady state conditions prevail. The temperature of the end B is then suddenly reduced to $0^\circ c$ and that of the end A is maintained. Find the subsequent temperature distribution u(x, t) at a distance *x* at time *t*. (Dec 2010/Dec 2014/May 2014)

Solution:

We know that the heat flow equation is $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

Given $\theta_1=0$ *and* $\theta_2=100$

w.k.t $u=\left(\frac{\theta_2-\theta_1}{\ell}\right)x+\theta_1$

$$u=\left(\frac{100-0}{\ell}\right)x+0$$

$$u=\left(\frac{100}{\ell}\right)x$$

Now the boundary conditions are

(i) $u(0,t)=0,$ for all $t>0$

(ii) $u(l,t)=0,$ for all $t>0$

(iii) $u(x,0)=\frac{100x}{l},\ 0<x<\ell$

The suitable solution of $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

$$u(x,t)=(A\cos px+B\sin px)e^{-\alpha^2p^2t} \qquad \text{----(1)}$$

Applying condition (i) in eqn (1) we get

$$u(0,t) = Ae^{-\alpha^2 p^2 t} = 0$$

$$But \;\; e^{-\alpha^2 p^2 t} \neq 0 \qquad \therefore A = 0$$

Substituting $A = 0$ in (4) we get

$$u(x,t) = B\sin px e^{-\alpha^2 p^2 t} = 0 \qquad \text{----(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t) = B\sin pl e^{-\alpha^2 p^2 t} = 0$$

Here $B \neq 0$ since if $B = 0$ we get trivial solution.

$e^{-\alpha^2 p^2 t} \neq 0$ since it is true for all 't'

Hence $\sin pl = 0$ i.e., $pl = n\pi$

Therefore $p = \dfrac{n\pi}{l}$

Substituting $p = \dfrac{n\pi}{l}$ in (2) we get

$$u(x,t) = B\sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}} \qquad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = \frac{100x}{l}$$

To find B_n**:** expand $\dfrac{100x}{l}$ in $(0,l)$ in half- range Fourier sine series we get,

$$B_n = \frac{2}{l}\int_0^l \frac{100x}{l}\sin\frac{n\pi x}{l}dx$$

$$= \frac{200}{l^2}\left[x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - 1\left(\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l = \frac{200}{l^2}\left[\frac{-l^2\cos n\pi}{n\pi}\right]$$

$$= \frac{200}{n\pi}(-1)^{n+1} \left[\because \cos n\pi = (-1)^n\right] \text{---------- (4)}$$

Substituting (4) in (3) we get

$$u(x,t)=\sum_{n=1}^{\infty}\frac{200}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2 n^2\pi^2 t}{l^2}}$$

6. A rod of length '*l*' has its ends kept at $0^\circ c$ and $200^\circ c$ respectively, until steady state conditions prevail. The temperature of the end B is then suddenly reduced to $0^\circ c$ and that of the end A is maintained. Find the subsequent temperature distribution u(x, t) at a distance *x* from the point A and at time *t*. (Dec 2012)

Solution:

We know that the heat flow equation is $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

Given $\theta_1=0$ *and* $\theta_2=200$

w.k.t $u=\left(\frac{\theta_2-\theta_1}{\ell}\right)x+\theta_1$

$$u=\left(\frac{200-0}{\ell}\right)x+0$$

$$u=\frac{200}{\ell}x$$

Now the boundary conditions are

$(i)\ u(0,t)=0,\quad \text{for all}\quad t>0$

$(ii)\ u(l,t)=0,\quad \text{for all}\quad t>0$

$(iii)\ u(x,0)=\frac{200x}{l},\ 0<x<\ell$

The suitable solution of $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ is

$$u(x,t)=(A\cos px+B\sin px)e^{-\alpha^2 p^2 t} \qquad \text{----(1)}$$

Applying condition (i) we get

$$u(0,t)=Ae^{-\alpha^2 p^2 t}=0$$

$$But\ \ e^{-\alpha^2 p^2 t}\neq 0 \qquad \therefore A=0$$

Substituting $A=0$ in (1) we get

$$u(x,t)=B\sin pxe^{-\alpha^2 p^2 t}=0 \qquad \text{----(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t) = B\sin pl e^{-\alpha^2 p^2 t} = 0$$

Here $B \neq 0$ since if $B = 0$ we get trivial solution.

$e^{-\alpha^2 p^2 t} \neq 0$ since it is true for all 't'

Hence $\sin pl = 0$ i.e., $pl = n\pi$

Therefore $p = \dfrac{n\pi}{l}$

Substituting $p = \dfrac{n\pi}{l}$ in (2) we get

$$u(x,t) = B\sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}} \qquad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = \frac{200x}{l}$$

To find B_n expand $\dfrac{200x}{l}$ in $(0,l)$ in half- range Fourier sine series we get,

$$\frac{200x}{l} = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$$

Now $B_n = \dfrac{2}{l}\displaystyle\int_0^l \frac{200x}{l}\sin\frac{n\pi x}{l}\,dx$

$$= \frac{400}{l^2}\left[x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - 1\left(\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l = \frac{400}{l^2}\left[\frac{-l^2\cos n\pi}{n\pi}\right]$$

$$= \frac{400}{n\pi}(-1)^{n+1} \left[\because \cos n\pi = (-1)^n\right]$$

Substituting in (3) we get

$$u(x,t) = \sum_{n=1}^{\infty} \frac{400}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}$$

7.A rod 30 cm, long has its ends A and B kept at $20^\circ c$ and $80^\circ c$ respectively. Until steady

state conditions prevail. The temperature at each end is then suddenly reduced to $0^{\circ}c$ and kept so. Find the temperature u(x, t). (Dec2011)

Solution:

We know that the heat flow equation is $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

Given $\theta_1 = 20$ *and* $\theta_2 = 80$ *and* $\ell = 30$

w.k.t $u = \left(\frac{\theta_2 - \theta_1}{\ell}\right)x + \theta_1$

$$u = \left(\frac{80-20}{30}\right)x + 20$$

$$u = 2x + 20$$

Now the boundary conditions are

$(i)\ u(0,t) = 0,$ for all $t > 0$

$(ii)\ u(l,t) = 0,$ for all $t > 0$

$(iii)\ u(x,0) = 2x + 20,\ 0 < x < \ell$

The suitable solution of $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ is

$$u(x,t) = (A\cos px + B\sin px)e^{-\alpha^2 p^2 t} \qquad \text{---------- (1)}$$

Applying condition (i) we get

$$u(0,t) = Ae^{-\alpha^2 p^2 t} = 0$$

$$But\ \ e^{-\alpha^2 p^2 t} \neq 0 \qquad \therefore A = 0$$

Substituting $A = 0$ in (4) we get

$$u(x,t) = B\sin pxe^{-\alpha^2 p^2 t} = 0 \qquad \text{----------(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t) = B\sin ple^{-\alpha^2 p^2 t} = 0$$

Here $B \neq 0$ since if $B = 0$ we get trivial solution.

$e^{-\alpha^2 p^2 t} \neq 0$ since it is true for all 't'

Hence $\sin pl = 0$ i.e., $pl = n\pi$

Therefore $p = \frac{n\pi}{l}$

Substituting $p = \frac{n\pi}{l}$ in (2) we get

$$u(x,t) = B\sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}} \qquad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = \frac{60x}{l} + 20 \qquad \text{----(4)}$$

To find B_n expand f(x) in $(0,l)$ in half- range Fourier sine series we get,

$$2x + 20 = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$$

Now $B_n = \frac{2}{l}\int_0^l (2x+20)\sin\frac{n\pi x}{l}\,dx = \frac{2}{30}\int_0^{30}(2x+20)\sin\frac{n\pi x}{30}\,dx$, since $\ell = 30$

$$= \frac{2}{30}\left[(2x+20)\left(\frac{-\cos\frac{n\pi x}{30}}{\frac{n\pi}{30}}\right) - (2)\left(\frac{-\sin\frac{n\pi x}{30}}{\left(\frac{n\pi}{30}\right)^2}\right)\right]_0^{30}$$

$$= \frac{2}{30}\left[(80)\left(\frac{-\cos n\pi}{\frac{n\pi}{30}}\right) - (20)\left(\frac{-1}{\frac{n\pi}{30}}\right)\right] = \frac{40}{n\pi}\left[1 - 4(-1)^n\right]$$

Substituting in (4) we get

$$u(x,t) = \sum_{n=1}^{\infty} \frac{40}{n\pi}\left[1 - 4(-1)^n\right]\sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}}$$

STEADY STATE CONDITION AND NON – ZEROBOUNDARY CONDITION

1. **A rod of 30 cm long as its end a and b are at 20°c and 80 c respectively, until the steady state condition prevails .the temperature at the end b is suddenly reduced to 60 c and at end a suddenly raised to 40 c and maintained .find the temperature u(x,t).**

Solution:

The one dimensional heat equation is $\frac{\partial^2 u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

Here there are two states

$$i.e \quad u(x,t) = u_1(x) + u_2(x,t) \;\; ------\rightarrow (1)$$

Steady State I	Steady State II
$x=0$ $\quad$ $x=30$ $u=20$ $\quad$ $u=80$ The steady state solution is $u(x,0)=\left(\frac{\theta_2-\theta_1}{\ell}\right)x+\theta_1$ Here $\theta_1=20,\quad \theta_2=80 \;\; and \;\; \ell=30$ $u(x,0)=\left(\frac{80-20}{30}\right)x+20$ $u(x,0)=\frac{60}{30}x+20$ $u(x,0)=2x+20$ Now the B.C Are 1) $u(0,t)=40^{\circ}c, \forall t>0$ 2) $u(30,t)=60^{\circ}c, \forall t>0$ 3) $u(x,0)=2x+20, 0<x<30$	$x=0$ $\quad$ $x=30$ $u=40$ $\quad$ $u=60$ The steady state solution is $u(x,0)=\left(\frac{\theta_2-\theta_1}{\ell}\right)x+\theta_1$ Here $\theta_1=40,\quad \theta_2=60 \;\; and \;\; \ell=30$ $u_1(x)=\left(\frac{\theta_2-\theta_1}{l}\right)x+\theta_1$ $=\frac{60-40}{30}x+40$ $u_1(x)=\frac{2}{3}x+40$ Now the B.C Are $(i)\;\; u_1(0)=40^{\circ}c$ $(ii)\;\; u_1(30)=60^{\circ}c$ $(iii)\;\; u_1(x)=\frac{2}{3}x+40$

To find $u_2(x,t)$:

But $\quad u_2(x,t)=u(x,t)-u_1(x)$

$$(i)\;\; u_2(0,t)=u(0,t)-u_1(0) \;=\; 40^{\circ}-40^{\circ}=0$$

$$(ii)\;\; u_2(30,t)=u(30,t)-u_1(30) \;=60^{\circ}-60^{\circ}=0$$

$$(iii)\;\; u_2(x,0)=u(x,0)-u_1(x) \;=\; 2x+20-\frac{2}{3}x-40$$

$$\therefore\ u_2(x,t) = \frac{4}{3}x - 20$$

Now the boundary condition are

$i)\ \ u_2(0,t) = 0^{\circ}c,\ \ \forall t > 0$

$ii)\ \ u_2(30,t) = 0^{\circ}c,\ \ \forall t > 0$

$iii)\ \ u_2(x,0) = \frac{4}{3}x - 20,\ \ 0 < x < 30$

The suitable solution are,

$$u_2(x,t) = [A\cos px + B\sin px]Ce^{-\alpha^2 p^2 t}\ \ ----\rightarrow (2)$$

Apply (i) boundary condition in (2)

$u_2(0,t) = [A\cos 0 + B\sin 0]Ce^{-\alpha^2 p^2 t}$

$i.e)\ \ ACe^{-\alpha^2 p^2 t} = 0$

But $\qquad Ce^{-\alpha^2 p^2 t} \neq 0$

$\therefore\ \ A = 0$

Substitute in equation (2)

$u_2(x,t) = B\sin pxCe^{-\alpha^2 p^2 t}\ \ -----\rightarrow (3)$

Apply (ii) boundary condition in (3)

$u_2(30,t) = B\sin 30p\ Ce^{-\alpha^2 p^2 t} = 0$

$Since\ \ B \neq 0\ \ and\ \ Ce^{-\alpha^2 p^2 t} \neq 0$

$\therefore\ \sin 30p = 0$

$30p = \sin^{-1}(0)$

$30p = n\pi$

$p = n\pi / 30$

Substitute in equation (3)

$$u_2(x,t) = B\sin\left(\frac{n\pi x}{30}\right)Ce^{-\alpha^2\frac{n^2\pi^2}{30^2}t}$$

$$u_2(x,t) = B_n\sin\left(\frac{n\pi x}{30}\right)e^{-\alpha^2\frac{n^2\pi^2}{30^2}t}$$

The most general solution can be written as

$$\boxed{u_2(x,t) = \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{30}\right)e^{-\alpha^2\frac{n^2\pi^2}{30^2}t}} \text{-------}\rightarrow (4)$$

Apply (iii) boundary condition in (4)

$$u_2(x,0) = \sum_{n=1}^{\infty} B_n\sin\frac{n\pi}{30}xe^{-\alpha^2\frac{n^2\pi^2}{30^2}(0)} = f(x)$$

$$\sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{30}\right) = \frac{4}{3}x - 20$$

To find B_n :

$$B_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$$

$$B_n = \frac{2}{30}\int_0^{30}\left(\frac{4}{3}x-20\right)\sin\left(\frac{n\pi x}{30}\right)dx$$

$$= \frac{2}{30}\left[\left(\frac{4}{3}x-20\right)\left(\frac{-\cos\left(\frac{n\pi x}{30}\right)}{\frac{n\pi}{30}}\right)-\left(\frac{4}{3}\right)\left(\frac{-\sin\left(\frac{n\pi x}{30}\right)}{\frac{n^2\pi^2}{30^2}}\right)\right]_0^{30}$$

$$= \frac{2}{30}\left[\frac{-20(30)}{n\pi}\cos(n\pi)+\frac{(-20)(30)}{n\pi}\cos 0\right]$$

$$= \frac{2}{30}\times\frac{-(20)(30)}{n\pi}\left[(-1)^n+1\right]$$

$$= \frac{-40}{n\pi}\left[(-1)^n+1\right]$$

$$=\begin{cases} 0, & if \;\; n \; is \;\; odd \\ \dfrac{-80}{n\pi}, & if \;\; n \; is \; even \end{cases}$$

Substitute in equation (3)

$$u_2(x,t)=\frac{-80}{\pi}\sum_{n=even}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{30}\right)e^{-\alpha^2\frac{n^2\pi^2}{900}t}$$

But $\quad u(x,t)=u_1(x)+u_2(x,t)$

$$u(x,t)=\frac{2}{3}x+40-\frac{80}{\pi}\sum_{n=even}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{30}\right)e^{-\alpha^2\frac{n^2\pi^2}{900}t}$$

2. Two ends a and b of a rod of length 20 cm as the temperature 30 c and 80 c respectively until the steady state condition prevails then the temperature at the a and b are changed into 40 c and 60 c. Find u(x,t)

Solution:

The one dimensional heat equation is $\dfrac{\partial^2 u}{\partial t}=\alpha^2\dfrac{\partial^2 u}{\partial x^2}$

Here there are two states

$$i.e \quad u(x,t)=u_1(x)+u_2(x,t) \;\; ------\rightarrow (1)$$

Steady State I	Steady State II
$x=0 \qquad x=20$ $u=30 \qquad u=80$ The steady state solution is $u(x,0)=\left(\dfrac{\theta_2-\theta_1}{\ell}\right)x+\theta_1$ Here $\theta_1=30, \quad \theta_2=80 \;\; and \;\; \ell=20$ $u(x,0)=\left(\dfrac{80-30}{20}\right)x+30$ $u(x,0)=\dfrac{50}{20}x+30$ $u(x,0)=\dfrac{5}{2}x+30$ Now the B.C Are	$x=0 \qquad x=20$ $u=40 \qquad u=60$ The steady state solution is $u(x,0)=\left(\dfrac{\theta_2-\theta_1}{\ell}\right)x+\theta_1$ Here $\theta_1=40, \quad \theta_2=60 \;\; and \;\; \ell=20$ $u_1(x)=\left(\dfrac{\theta_2-\theta_1}{l}\right)x+\theta_1$ $=\dfrac{60-40}{20}x+40$ $u_1(x)=x+40$ Now the B.C Are

1) $u(0,t)=40^{\circ}c, \forall t>0$ 2) $u(30,t)=60^{\circ}c, \forall t>0$ 3) $u(x,0)=\frac{5}{2}x+30, \quad 0<x<20$	$(i)\ u_1(0)=40^{\circ}c$ $(ii)\ u_1(30)=60^{\circ}c$ $(iii)\ u_1(x)=x+40$

To find $u_2(x,t)$:

But $\quad u_2(x,t)=u(x,t)-u_1(x)$

$$(i)\ u_2(0,t)=u(0,t)-u_1(0) = 40^{\circ}-40^{\circ}=0$$

$$(ii)\ u_2(20,t)=u(20,t)-u_1(20) = 60^{\circ}-60^{\circ}=0$$

$$(iii)\ u_2(x,0)=u(x,0)-u_1(x) = \frac{5}{2}x+30-x-40$$

$$\therefore\ u_2(x,t)=\frac{3}{2}x-10$$

Now the boundary condition are

$i)\ u_2(0,t)=0^{\circ}c, \quad \forall t>0$

$ii)\ u_2(20,t)=0^{\circ}c, \quad \forall t>0$

$iii)\ u_2(x,0)=\frac{3}{2}x-10, \quad 0<x<20$

The suitable solution are,

$$u_2(x,t)=[A\cos px+B\sin px]Ce^{-\alpha^2p^2t} \quad ------\rightarrow (2)$$

Apply (i) boundary condition in (2)

$u_2(0,t)=[A\cos 0+B\sin 0]Ce^{-\alpha^2p^2t}$

$i.e)\quad ACe^{-\alpha^2p^2t}=0$

But $\quad Ce^{-\alpha^2p^2t}\neq 0$

$\therefore\ A=0$

Substitute in equation (2)

$u_2(x,t) = B\sin pxCe^{-\alpha^2 p^2 t} \quad -----\rightarrow (3)$

Apply (ii) boundary condition in (3)

$u_2(20,t) = B\sin 20p\ Ce^{-\alpha^2 p^2 t} = 0$

$Since \quad B \neq 0 \ \ and \quad Ce^{-\alpha^2 p^2 t} \neq 0$

$\therefore\ \sin 20p = 0$

$20p = \sin^{-1}(0)$

$20p = n\pi$

$p = n\pi / 20$

Substitute in equation (3)

$$u_2(x,t) = B\sin\left(\frac{n\pi x}{20}\right)Ce^{-\alpha^2 \frac{n^2\pi^2}{20^2}t}$$

$$u_2(x,t) = B_n \sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2 \frac{n^2\pi^2}{20^2}t}$$

The most general solution can be written as

$$\boxed{u_2(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2 \frac{n^2\pi^2}{20^2}t}} \text{-------}\rightarrow (4)$$

Apply (iii) boundary condition in (4)

$$u_2(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{20}xe^{-\alpha^2 \frac{n^2\pi^2}{20^2}(0)} = f(x)$$

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{20}\right) = \frac{3}{2}x - 10$$

To find B_n :

$$B_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$$

$$B_n = \frac{2}{20}\int_0^{20}\left(\frac{3}{2}x - 10\right)\sin\left(\frac{n\pi x}{20}\right)dx$$

$$=\frac{1}{10}\left[\left(\frac{3}{2}x-10\right)\left(\frac{-\cos\left(\frac{n\pi x}{20}\right)}{\frac{n\pi}{20}}\right)-\left(\frac{3}{2}\right)\left(\frac{-\sin\left(\frac{n\pi x}{20}\right)}{\frac{n^2\pi^2}{20^2}}\right)\right]_0^{20}$$

$$=\frac{1}{10}\left[\frac{-20(20)}{n\pi}\cos(n\pi)+\frac{(-20)(10)}{n\pi}\cos 0\right]$$

$$=\frac{1}{10}\times\frac{-(20)(10)}{n\pi}\left[2(-1)^n+1\right]$$

$$=\frac{-20}{n\pi}\left[2(-1)^n+1\right]$$

Substitute in equation (3)

$$u_2(x,t)=\frac{-20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[2(-1)^n+1\right]\sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2\frac{n^2\pi^2}{400}t}$$

But $\quad u(x,t)=u_1(x)+u_2(x,t)$

$$u(x,t)=x+40-\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[2(-1)^n+1\right]\sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2\frac{n^2\pi^2}{400}t}$$

<u>**TWO DIMENSIONAL HEAT EQUATION**</u>:

<u>**INFINITE PLATE**</u>

1. **Derive the two dimensional heat equation**:
   Solution:Solution:

The two dimensional heat equation is $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ -------(1)

Let $\quad u=XY$ ---------- (2)

$$\frac{\partial u}{\partial x}=X'Y$$

$$\frac{\partial u}{\partial y}=X\,Y'$$

$$\frac{\partial^2 u}{\partial x^2}=X''Y$$

$$\frac{\partial^2 u}{\partial y^2} = X\,Y''$$

Substitute in equation (1)

$$X''\,Y + X\,Y'' = 0$$

$$\frac{X''}{X} = \frac{-Y''}{Y} = k$$

Equating

Consider $\frac{X''}{X} = k, \qquad \frac{Y''}{Y} = -k$

$$X'' = kX, \quad and \quad Y'' = -kY$$

$$X'' - kX = 0 \quad -----(3)$$

$$Y'' + kY = 0 ----(4)$$

The solution of equation (3) and (4) depends up on k

Case (1):

Let k be positive

i.e k = p^2

$$(3) \Rightarrow \quad X'' - p^2 X = 0$$

The auxiliary equation is

$$m^2 - p^2 = 0$$

$$m^2 = p^2$$

$$m = \pm p$$

$\therefore$ The roots are $X = A_1 e^{px} + A_2 e^{-px} \quad -----(5)$

$$(4) \Rightarrow \quad Y'' + p^2 Y = 0$$

The auxiliary equation is

$$m^2 + p^2 = 0$$

$$m^2 = -p^2$$

$$m = \pm pi$$

$\therefore$ The roots are $Y = A_3 \cos py + A_4 \sin py \quad -----(6)$

Substitute equation (5) and (6) in (2)

$$u(x,y) = \left[A_1 e^{px} + A_2 e^{-px}\right]\left[A_3 \cos py + A_4 \sin py\right] -----(I)$$

Case (2):

Let k be negative

i.e k = - p^2

$$(3) \Rightarrow \quad X'' + p^2 X = 0$$

The auxiliary equation is

$$m^2 + p^2 = 0$$

$$m^2 = -p^2$$

$$m = \pm pi$$

$\therefore$ The roots are $X = A_5 \cos px + A_6 \sin px \quad -----(7)$

$$(4) \Rightarrow \quad Y'' - p^2 Y = 0$$

The auxiliary equation is

$$m^2 - p^2 = 0$$

$$m^2 = p^2$$

$$m = \pm p$$

$\therefore$ The roots are $Y = A_7 \; e^{py} + A_8 \, e^{-py} \quad -----(8)$

Substitute equation (7) and (8) in (2)

$$u(x, y) = \left[A_5 \cos px + A_6 \sin px \right] \left[A_7 \; e^{py} + A_8 \, e^{-py} \right] -----(II)$$

Case (3):

Let k be zero

i.e k = 0

$$(3) \Rightarrow \quad X'' = 0$$

The auxiliary equation is

$$m^2 = 0$$

$$m = 0, 0$$

$\therefore$ The roots are $X = A_9 + A_{10} x \quad -----(9)$

$$(4) \Rightarrow \quad Y'' = 0$$

The auxiliary equation is

$$m^2 = 0$$

$$m = 0, 0$$

$\therefore$ The roots are $Y = A_{11} + A_{12}y \quad -----(10)$

Substitute equation (9) and (10) in (2)

$$u(x,y) = \left[A_9 + A_{10}x\right]\left[A_{11} + A_{12}y\right] -----(III)$$

Thus the various possible solutions are

i) $u(x,y) = \left[A_1 e^{px} + A_2 e^{-px}\right]\left[A_3 \cos py + A_4 \sin py\right] -----(I)$

ii) $u(x,y) = \left[A_5 \cos px + A_6 \sin px\right]\left[A_7\, e^{py} + A_8\, e^{-py}\right] -----(II)$

iii) $u(x,y) = \left[A_9 + A_{10}x\right]\left[A_{11} + A_{12}y\right] -----(III)$

NOTE:

1. If the non – zeroboundary condition lies on x – axisand parallel to x – axis,then choose the solution [ii]
2. If the non – zeroboundary condition lies on y – axis and parallel to y – axis,then choose the solution [i]

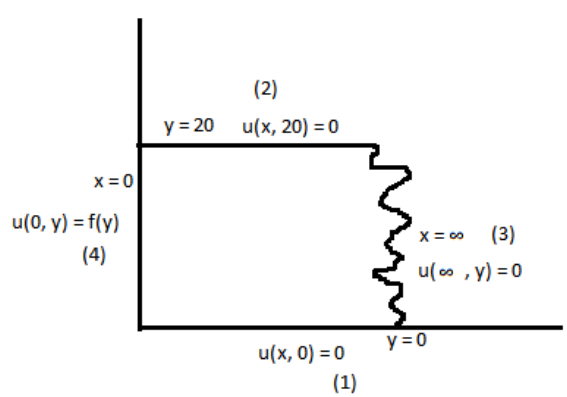
PROBLEMS:

1. **A rectangular plate with insulated surface wide and so long compared to its width it may be considered infinite in length ,with introducing an revisable error. If the temperature of the short edge x=0 is given** by $u = \begin{cases} 10\,y, & 0 \le y \le 10 \\ 10(20-y), & 10 \le y \le 20 \end{cases}$ **And the two long edges as well as the other short edge are kept at 0°c.Find the steady state temperature distribution in the plate.**

Solution:

The two dimensional equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$



The boundary conditions are

1) $u(x,0)=0, \quad 0<x<\infty$

2) $u(x,20)=0, \quad 0<x<\infty$

3) $u(\infty,y)=0, \quad 0<y<20$

4) $u(0,y)=f(y)=\begin{cases}10y, & 0\le y\le 10\\ 10(20-y), & 10\le y\le 20\end{cases}$

Now the possible solutions (trigonometry function in y)as

$$u(x,y)=\left[Ae^{px}+Be^{-px}\right]\left[C\cos py+D\sin py\right] -----(1)$$

Apply 1st Boundary Conditions in [1]

$$u(0,y)=\left[Ae^{px}+Be^{-px}\right]\left[C(1)+D(0)\right]=0$$

$$u(0,y)=\left[Ae^{px}+Be^{-px}\right]C=0$$

But $\left[Ae^{px}+Be^{-px}\right]\neq 0$

$$\therefore \; C=0$$

Sub c=0 in [1]

$$u(x,y)=\left[Ae^{px}+Be^{-px}\right]D\sin py \; -----(2)$$

Apply 2nd Boundary Condition in [2]

$$u(x,20)=\left[Ae^{px}+Be^{-px}\right]D\sin 20p=0$$

But $\left[Ae^{px}+Be^{-px}\right]\neq 0 \;\; and \;\; D\neq 0$

$$\therefore \; \sin 20p=0$$

$$20p=\sin^{-1}(0)=n\pi$$

$$p=\frac{n\pi}{20}$$

Sub p value in [2]

$$u(x,y)=\left[Ae^{\left(\frac{n\pi x}{20}\right)}+Be^{-\left(\frac{n\pi x}{20}\right)}\right]D\sin\left(\frac{n\pi y}{20}\right)\ -----(3)$$

Apply 3rd Boundary Condition in [3]

$As \quad x\to\infty \ \Rightarrow \ A=0$

$\therefore A=0$

Sub A=0 in [3]

$$u(x,y)=Be^{-\left(\frac{n\pi x}{20}\right)}D\sin\left(\frac{n\pi y}{20}\right)$$

$$u(x,y)=B_n\,e^{-\left(\frac{n\pi x}{20}\right)}\sin\left(\frac{n\pi y}{20}\right)$$

The most general solution is

$$\boxed{u(x,y)=\sum_{n=1}^{\infty}B_n\,e^{-\left(\frac{n\pi x}{20}\right)}\sin\left(\frac{n\pi y}{20}\right)}\ -------\rightarrow(4)$$

Apply 4thBoundary Condition in (4)

$$u(0,y)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi y}{20}\right)=f(y)=\begin{cases}10y, & 0\le y\le 10\\ 10(20-y), & 10\le y\le 20\end{cases}$$

To find B_n: Expand Fourier sine half range series

$$B_n=\frac{2}{l}\int_0^{\ell}f(x)\sin\frac{n\pi x}{l}$$

$$=\frac{2}{20}\left(\int_0^{10}10y\sin\left(\frac{n\pi x}{20}\right)dx+\int_{10}^{20}10(20-y)\sin\left(\frac{n\pi x}{20}\right)dx\right)$$

$$=\frac{10}{10}\left[y\left(-\frac{\cos\frac{n\pi y}{20}}{\frac{n\pi}{20}}\right)-\left(-\frac{\sin\frac{n\pi y}{20}}{\frac{n^2\pi^2}{20^2}}\right)\right]_0^{10}+\left[(20-y)\left(-\frac{\cos\frac{n\pi y}{20}}{\frac{n\pi}{20}}\right)+\left(-\frac{\sin\frac{n\pi y}{20}}{\frac{n^2\pi^2}{20^2}}\right)\right]_{10}^{20}$$

$$=\left[\frac{-10(20)}{n\pi}\cos\left(\frac{n\pi}{2}\right)+\frac{400}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)+\frac{10(20)}{n\pi}\cos\left(\frac{n\pi}{2}\right)+\frac{400}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{800}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$$B_n = \frac{800}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

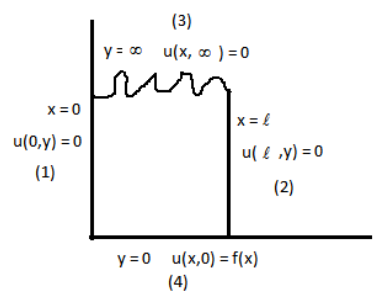
Substitute in equation (4)

$$u(x,y) = \sum_{n=1}^{\infty}\frac{800}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\ e^{-\left(\frac{n\pi x}{20}\right)}\sin\left(\frac{n\pi y}{20}\right)$$

$$u(x,y) = \frac{800}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\ e^{-\left(\frac{n\pi x}{20}\right)}\sin\left(\frac{n\pi y}{20}\right)$$

2. **A long rectangular plate with insulated surface with l cm wide. If the temperature of the one short edge y=0 is given by** $u(x,0) = k\left(\ell x - x^2\right),\ \ o < x < \ell$ **while the other three edges are kept at 0°c.Find the steady state temperature distribution in the plate.**

Solution:



The two dimensional equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

The boundary conditions are

1) $u(0,y) = 0,\ \ 0 < y < \infty$

2) $u(\ell,y) = 0,\ \ 0 < y < \infty$

3) $u(x,\infty) = 0,\ \ 0 < x < \ell$

4) $u(x,0) = f(x) = k\left(\ell x - x^2\right),\ \ o < x < \ell$

Now the possible solutions (trigonometry function in x)as

$$u(x,y)=\left[A\cos px+B\sin px\right]\left[Ce^{py}+De^{-py}\right]-----(1)$$

Apply 1st Boundary Conditions in [1]

$$u(0,y)=\left[A(1)+B(0)\right]\left[Ce^{py}+De^{-py}\right]=0$$

$$u(0,y)=A\left[Ce^{py}+De^{-py}\right]=0$$

But $\left[Ce^{py}+De^{-py}\right]\neq 0$

$$\therefore\ A=0$$

Sub A=0 in [1]

$$u(x,y)=B\sin px\left[Ce^{py}+De^{-py}\right]-----(2)$$

Apply 2nd Boundary Condition in [2]

$$u(\ell,y)=B\sin p\ell\left[Ce^{py}+De^{-py}\right]=0$$

But $B\neq 0\ \ and\ \ \left[Ce^{py}+De^{-py}\right]\neq 0$

$$\therefore\ \sin p\ell=0$$

$$p\ell=\sin^{-1}(0)=n\pi$$

$$p=\frac{n\pi}{\ell}$$

Sub p value in [2]

$$u(x,y)=B\sin\left(\frac{n\pi x}{\ell}\right)\left[Ce^{\left(\frac{n\pi y}{\ell}\right)}+De^{-\left(\frac{n\pi y}{\ell}\right)}\right]-----(3)$$

Apply 3rd Boundary Condition in [3]

$$As\quad y\to\infty\ \Rightarrow\ C=0$$

$$\therefore C=0$$

Sub C=0 in [3]

$$u(x,y) = B\sin\left(\frac{n\pi x}{\ell}\right) D e^{-\left(\frac{n\pi y}{\ell}\right)}$$

$$u(x,y) = B_n \sin\left(\frac{n\pi x}{\ell}\right) e^{-\left(\frac{n\pi y}{\ell}\right)}$$

The most general solution is

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{\ell}\right) e^{-\left(\frac{n\pi y}{\ell}\right)}} \longrightarrow (4)$$

Apply 4thBoundary Condition in (4)

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{\ell}\right) = f(x) = k\left(\ell x - x^2\right),\ 0 < x < \ell$$

To find B_n: Expand Fourier sine half range series

$$B_n = \frac{2}{l}\int_0^{\ell} f(x)\sin\frac{n\pi x}{l}$$

$$B_n = \frac{2}{l}\int_0^{l} k(lx - x^2)\sin\frac{n\pi x}{l}\, dx$$

$$= \frac{2k}{l}\left[(lx - x^2)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (l - 2x)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + (-2)\left(\frac{\cos\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right)\right]_0^{\ell}$$

$$= \frac{2k}{l}\left[0 - 2\left(\frac{\cos n\pi}{\frac{n^3\pi^3}{l^3}}\right) - 0 + 2\left(\frac{\cos 0}{\frac{n^3\pi^3}{l^3}}\right)\right] \qquad [$$

$\therefore \quad \sin n\pi = 0$]

$$= \frac{2k}{\ell} \times \frac{2\ell^3}{n^3\pi^3}\left[-(-1)^n + 1\right]$$

$$B_n = \frac{4kl^2}{n^3\pi^3}\left[1-(-1)^n\right]$$

$$B_n = \begin{cases} \dfrac{8kl^2}{n^3\pi^3}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

Substituting B_n in (4) we get

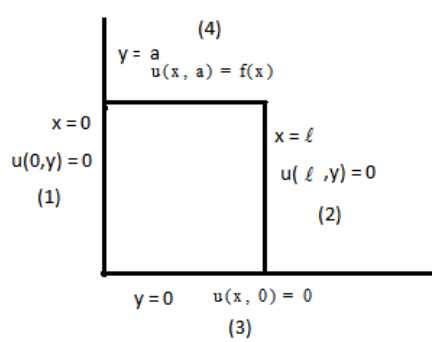
$$u(x,y) = \sum_{n=1}^{\infty} \frac{8kl^2}{n^3\pi^3} \sin\left(\frac{n\pi x}{\ell}\right) e^{-\left(\frac{n\pi y}{\ell}\right)}$$

FINITE LENGTH OF THE PLATE:

1. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ **subject to the condition u (0, y) = u(l, y) = u(x, 0) and u(x, a) =** $\sin\left(\frac{n\pi x}{l}\right)$

Solution:

The two dimensional heat equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$



The boundary conditions are,

i) u(0, y) = 0,0<y<a
ii) $u(\ell, y)$ = 0,0<y<a
iii) u(x,0)=0, 0<x<l

iv) u(x,a) = f(x)= $\sin\left(\frac{n\pi x}{l}\right)$,0<x<l

The suitable boundary conditions are,

$$u(x,y)=\left[A\cos px+B\sin px\right]\left[Ce^{py}+De^{-py}\right]-----(1)$$

Apply first boundary condition in (1)

$$u(0,y)=\left[A(1)+B(0)\right]\left[Ce^{py}+De^{-py}\right]=0$$

Here, $\left[Ce^{py}+De^{-py}\right]\neq 0$

$\therefore \quad A=0$

Subs. A=0 in (1)

$$u(x,y)=B\sin px\left[Ce^{py}+De^{-py}\right]-----(2)$$

Apply second boundary condition in (2),

$$u(\ell,y)=B\sin p\ell\left[Ce^{py}+De^{-py}\right]=0$$

Here, $B\neq 0 \quad and \quad \left[Ce^{py}+De^{-py}\right]\neq 0$

$$\therefore \sin p\ell=0$$

$$p\ell=\sin^{-1}(0)=n\pi$$

$$p=\frac{n\pi}{l}$$

subs p value in (2),

$$u(x,y)=B\sin\left(\frac{n\pi x}{l}\right)\left[Ce^{\frac{n\pi y}{l}}+De^{-\frac{n\pi y}{l}}\right]-------(3)$$

Apply 3 rd boundary condition in (3)

$$u(x,0)=B\sin\left(\frac{n\pi x}{l}\right)\left[C+D\right]=0$$

B $\sin\frac{n\pi x}{l}\neq 0$

C+D = 0

C = -D

Sub c value in (3),

$$u(x,y) = B\sin\left(\frac{n\pi x}{l}\right)\left[Ce^{\frac{n\pi y}{l}} - Ce^{-\frac{n\pi y}{l}}\right]$$

$$u(x,y) = BC\sin\left(\frac{n\pi x}{l}\right)\left[e^{\frac{n\pi y}{l}} - e^{-\frac{n\pi y}{l}}\right]$$

$$u(x,y) = BC\sin\left(\frac{n\pi x}{l}\right)2\sinh\left(\frac{n\pi y}{l}\right)$$

$$u(x,y) = B_n\sin\left(\frac{n\pi x}{l}\right)\sinh\left(\frac{n\pi y}{l}\right), \quad where \;\; B_n = 2BC$$

The most general solution is ,

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{l}\right)\sinh\left(\frac{n\pi y}{l}\right)} -------(4)$$

Apply 4th Boundary condition in (4),

$$u(x,a) = \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{l}\right)\sinh\left(\frac{n\pi a}{l}\right) = f(x)$$

$$\sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{l}\right)\sinh\left(\frac{n\pi a}{l}\right) = \sin\left(\frac{n\pi x}{l}\right)$$

Equating n^{th} term on both sides

$$B_n\sinh\left(\frac{n\pi a}{l}\right) = 1$$

$$B_n = \frac{1}{\sinh\frac{n\pi a}{l}}$$

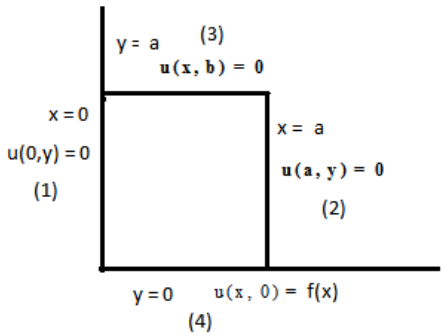
Subs in (5),

$$u(x,y) = \sum_{n=1}^{\infty}\frac{1}{\sinh\left(\frac{n\pi a}{l}\right)}\sin\left(\frac{n\pi x}{l}\right)\sinh\left(\frac{n\pi y}{l}\right)$$

2. **Solve for steady state temperature of any point of a rectangular plate of size A & B insulated on the lateral surface and satisfying u(0,y) = 0,u(a,y) = 0, u(x,b) = 0,u(x,0) = x(a- x)**

Solution:

The two dimensional heat equation is $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$



The boundary conditions are,

(i) u(0,y) = 0, 0<y<b
(ii) u(a,y)= 0, 0<y<b
(iii) u(x,b)=0, 0<x<a
(iv) u(x,0) = f(x)=x(a – x),0<x<a

The suitable boundary conditions are,

$$u(x,y)=\left[A\cos px+B\sin px\right]\left[Ce^{py}+De^{-py}\right]-----(1)$$

Apply first boundary condition in (1)

$$u(0,y)=\left[A(1)+B(0)\right]\left[Ce^{py}+De^{-py}\right]=0$$

Here, $\left[Ce^{py}+De^{-py}\right]\neq 0$

$\therefore \quad A=0$

Subs. A=0 in (1)

$$u(x,y)=B\sin px\left[Ce^{py}+De^{-py}\right]-----(2)$$

Apply second boundary condition in (2),

$$u(a,y)=B\sin pa\left[Ce^{py}+De^{-py}\right]=0$$

Here, $B\neq 0 \quad and \quad \left[Ce^{py}+De^{-py}\right]\neq 0$

$$\therefore \sin pa = 0$$

$$pa = \sin^{-1}(0) = n\pi$$

$$p = \frac{n\pi}{a}$$

subs p value in (2),

$$u(x,y) = B\sin\left(\frac{n\pi x}{a}\right)\left[Ce^{\frac{n\pi y}{a}} + De^{-\frac{n\pi y}{a}}\right] -------(3)$$

Apply 3 rd boundary condition in (3)

$$u(x,b) = B\sin\left(\frac{n\pi x}{a}\right)\left[Ce^{\frac{n\pi b}{a}} + De^{-\frac{n\pi b}{a}}\right] = 0$$

$$\text{B} \sin\frac{n\pi x}{l} \neq 0$$

$$\left[Ce^{\frac{n\pi b}{a}} + De^{-\frac{n\pi b}{a}}\right] = 0$$

$$Ce^{\frac{n\pi b}{a}} = -De^{-\frac{n\pi b}{a}} = k$$

$$C = ke^{-\frac{n\pi b}{a}} \quad \text{and} \quad D = -ke^{\frac{n\pi b}{a}}$$

Sub Cand D value in (3),

$$u(x,y) = B\sin\left(\frac{n\pi x}{a}\right)\left[ke^{-\frac{n\pi b}{a}}e^{\frac{n\pi y}{a}} - ke^{\frac{n\pi b}{a}}e^{-\frac{n\pi y}{a}}\right]$$

$$u(x,y) = Bk\sin\left(\frac{n\pi x}{a}\right)\left[e^{\frac{n\pi(y-b)}{a}} - e^{-\frac{n\pi(y-b)}{a}}\right]$$

$$u(x,y) = Bk\sin\left(\frac{n\pi x}{a}\right)\left[2\sinh\left\{\frac{n\pi}{a}(y-b)\right\}\right]$$

$$u(x,y) = B_n\sin\left(\frac{n\pi x}{a}\right)\sinh\left\{\frac{n\pi}{a}(y-b)\right\}, \quad where \;\; B_n = 2Bk$$

The most general solution is ,

$$\boxed{u(x,y)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi x}{a}\right)\sinh\left\{\frac{n\pi}{a}\left(y-b\right)\right\}}\text{--------}(4)$$

Apply 4th Boundary condition in (4),

$$u(x,0)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi x}{a}\right)\sinh\left\{\frac{n\pi}{a}\left(-b\right)\right\}=f(x)$$

$$\sum_{n=1}^{\infty}-B_n\sin\left(\frac{n\pi x}{a}\right)\sinh\left\{\frac{n\pi b}{a}\right\}=f(x)$$

Expand B_n in fourier sine series

$$-B_n\sinh\left\{\frac{n\pi b}{a}\right\}=f(x)=\frac{2}{a}\int_0^a x(a-x)\sin\left(\frac{n\pi x}{a}\right)dx$$

$$=\frac{2}{a}\int_0^a (ax-x^2)\sin\frac{n\pi x}{a}\ \text{dx}$$

$$=\frac{2}{a}\left[(ax-x^2)\left(\frac{-\cos\frac{n\pi x}{a}}{n\pi/a}\right)-(a-2x)\left(\frac{-\sin\frac{n\pi x}{a}}{(n\pi/a)^2}\right)-2\left(\frac{\cos\frac{n\pi x}{a}}{(n\pi/a)^3}\right)\right]_0^a$$

$$=\frac{2}{a}\left[-2\cos n\pi\times\frac{a^3}{n^3\pi^3}+2\frac{a^3}{n^3\pi^3}\right]$$

$$=\frac{2}{a}\left[2\frac{a^3}{n^3\pi^3}\left\{1-\left(-1\right)^n\right\}\right]$$

$$=\frac{4a^2}{n^3\pi^3}\left[1-\left(-1\right)^n\right]$$

$$-B_n\sinh\left\{\frac{n\pi b}{a}\right\}=\begin{cases}\dfrac{8a^2}{n^3\pi^3}, & n\ is\ \ odd\\ 0, & n\ is\ even\end{cases}$$

$$B_n=\begin{cases}\dfrac{-8a^2}{n^3\pi^3\sinh\left\{\frac{n\pi b}{a}\right\}}, & n\ is\ \ odd\\ 0, & n\ is\ even\end{cases}$$

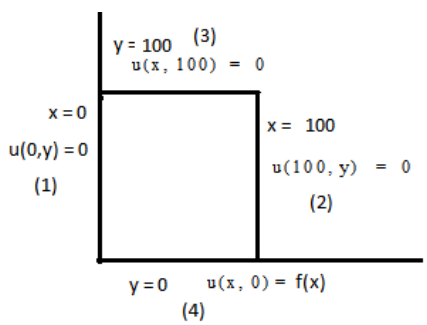
Subs B_n value in (4),

$$u(x,y)=\sum_{n=\,odd}^{\infty}\frac{-8a^2}{n^3\pi^3\sinh\left\{\frac{n\pi b}{a}\right\}}\sin\left(\frac{n\pi x}{a}\right)\sinh\left\{\frac{n\pi}{a}(y-b)\right\}$$

3. A square plate of side 100cm has a edge y=0 kept at 100° c and the other three edges kept at 0°c. Find the steady state temperature distribution.

Solution:

The two dimensional heat equation is $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$



The boundary conditions are,

(i) u(0,y) = 0, 0<y<100
(ii) u(100,y)= 0, 0<y<100
(iii) u(x,100)=0, 0<x<100
(iv) u(x, 0) = f(x)=100,0<x<100

The suitable boundary conditions are,

$$u(x,y)=\left[A\cos px+B\sin px\right]\left[Ce^{py}+De^{-py}\right]-----(1)$$

Apply first boundary condition in (1)

$$u(0,y)=\left[A(1)+B(0)\right]\left[Ce^{py}+De^{-py}\right]=0$$

Here, $\left[Ce^{py}+De^{-py}\right]\neq 0$

$\therefore\quad A=0$

Subs. A=0 in (1)

$$u(x,y) = B\sin px\left[Ce^{py} + De^{-py}\right] ------(2)$$

Apply second boundary condition in (2),

$$u(100,y) = B\sin p100\left[Ce^{py} + De^{-py}\right] = 0$$

Here, $B \neq 0 \quad and \quad \left[Ce^{py} + De^{-py}\right] \neq 0$

$$\therefore \sin p100 = 0$$

$$p100 = \sin^{-1}(0) = n\pi$$

$$p = \frac{n\pi}{100}$$

subs p value in (2),

$$u(x,y) = B\sin\left(\frac{n\pi x}{100}\right)\left[Ce^{\frac{n\pi y}{100}} + De^{-\frac{n\pi y}{100}}\right] -------(3)$$

Apply 3 rd boundary condition in (3)

$$u(x,100) = B\sin\left(\frac{n\pi x}{100}\right)\left[Ce^{n\pi} + De^{-n\pi}\right] = 0$$

$$\text{B} \sin\frac{n\pi x}{l} \neq 0$$

$$\left[Ce^{n\pi} + De^{-n\pi}\right] = 0$$

$$Ce^{n\pi} = -De^{-n\pi} = k$$

$$C = ke^{-n\pi} \quad \text{and} \quad D = -ke^{n\pi}$$

Sub C and D value in (3),

$$u(x,y) = B\sin\left(\frac{n\pi x}{100}\right)\left[ke^{-n\pi}e^{\frac{n\pi y}{100}} - ke^{n\pi}e^{-\frac{n\pi y}{100}}\right]$$

$$u(x,y) = Bk\sin\left(\frac{n\pi x}{100}\right)\left[e^{n\pi\left(\frac{y}{100}-1\right)} - e^{-n\pi\left(\frac{y}{100}-1\right)}\right]$$

$$u(x,y) = Bk\sin\left(\frac{n\pi x}{100}\right)\left[2\sinh\left\{n\pi\left(\frac{y}{100}-1\right)\right\}\right]$$

$$u(x,y) = B_n\sin\left(\frac{n\pi x}{100}\right)\sinh\left\{n\pi\left(\frac{y}{100}-1\right)\right\}, \quad where \;\; B_n = 2Bk$$

The most general solution is ,

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{100}\right)\sinh\left\{n\pi\left(\frac{y}{100}-1\right)\right\}} \text{--------}(4)$$

Apply 4th Boundary condition in (4),

$$u(x,0) = \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{100}\right)\sinh\left\{n\pi\left(\frac{y}{100}-1\right)\right\} = f(x)$$

$$\sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi x}{100}\right)\sinh\{n\pi\} = f(x)$$

Expand B_n in fourier sine series

$$B_n\sinh\{n\pi\} = f(x) = \frac{2}{100}\int_0^{100} 100\sin\left(\frac{n\pi x}{100}\right)dx$$

$$= \frac{200}{100}\int_0^{100}\sin\frac{n\pi x}{100}dx$$

$$= 2\left(\frac{-\cos\frac{n\pi x}{100}}{n\pi/100}\right)_0^{100}$$

$$= \frac{200}{n\pi}\left(\cos n\pi - \cos 0\right)$$

$$= \frac{200}{n\pi}\left[(-1)^n - 1\right]$$

$$B_n\sinh\{n\pi\} = \begin{cases} \frac{400}{n\pi}, & n \;\; is \;\; odd \\ 0, & n \;\; is \;\; even \end{cases}$$

$$B_n = \begin{cases} \dfrac{400}{n\pi \sinh\{n\pi\}}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

Subs B_n value in (4),

$$u(x,y) = \sum_{n=\text{odd}}^{\infty} \frac{400}{n\pi \sinh\{n\pi\}} \sin\left(\frac{n\pi x}{100}\right) \sinh\left\{n\pi\left(\frac{y}{100} - 1\right)\right\}$$

<u>Two Dimensional Heat equation in Polar Co-ordinates:</u>

The Three Possible Solutions of u(r,θ) are

(i) $u(r,\theta) = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta)$

(ii) $u(r,\theta) = (c_5 \cos(p \log r) + c_6 \sin(p \log r))(c_7^{\,p\theta} + c_8^{\,-p\theta})$

iii) $u(r,\theta) = (c_9 \log r + c_{10})(c_{11}\theta + c_{12})$

The Laplace equation in Polar Co-Ordinates is $r^2 \dfrac{\partial^2 u}{\partial r^2} + r\dfrac{\partial u}{\partial r} + \dfrac{\partial^2 u}{\partial \theta^2} = 0 \qquad \rightarrow (1)$

Where,u=u(r,θ)

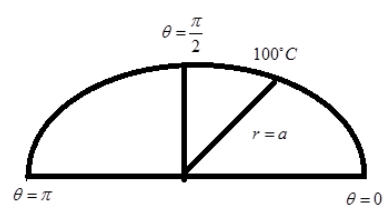
The solution of equation (1) is $u(r,\theta) = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta)$

1. **Find the steady state temp at any point of a semi-circular plate of radius 'a'. If the bounding diameter of the plate is kept at the temp 0°C and the circumference is kept at the temp 100°C.**

<u>Solution:</u>

The 2 dimensional heat equation in Polar Co-ordinates is,

$$r^2 \frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$$



The boundary conditions are,

1. u(r, 0) = 0, 0 < r < a
2. u(r, π) = 0, 0 < r < a
3. r→0, u→0, 0 < r < a

4. u(a, θ) = $f(\theta) = 100^{\circ}C$

The suitable solution which satisfy our boundary conditions are

$$u(r,\theta) = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta) ------\rightarrow (1)$$

<u>Apply 1st Boundary Condition in (1) :</u>

$$u(r,0) = (c_1 r^p + c_2 r^{-p})\left[c_3 \cos(0) + c_4 \sin(0)\right]$$

$$u(r,0) = (c_1 r^p + c_2 r^{-p})c_3 = 0$$

But $(c_1 r^p + c_2 r^{-p}) \neq 0$

$$\therefore c_3 = 0$$

Substitute in equation (1)

$$u(r,\theta) = (c_1 r^p + c_2 r^{-p})c_4 \sin p\theta ------\rightarrow (2)$$

<u>Apply 2nd Boundary Condition in (2):</u>

$$u(r,\pi) = (c_1 r^p + c_2 r^{-p})c_4 \sin p\pi = 0$$

But $(c_1 r^p + c_2 r^{-p}) \neq 0 \quad and \quad c_3 \neq 0$

$$\therefore \ \sin p\pi = 0$$

$$p\pi = \sin^{-1}(0)$$

$$p\pi = n\pi$$

$$\boxed{p = n}$$

Substitute in equation (1)

$$u(r,\theta) = (c_1 r^n + c_2 r^{-n})c_4 \sin n\theta ------\rightarrow (3)$$

<u>Apply 3rd Boundary Condition in (3):</u>

When $r \rightarrow 0 \quad \Rightarrow \quad c_2 \rightarrow 0$

Substitute c_2 value in eqn (3)

$$u(r,\theta) = c_1 r^n c_4 \sin n\theta$$

$$u(r,\theta) = B_n r^n \sin n\theta ------\rightarrow (4)$$

Themost general solution of eqn (4) is

$$u(r,\theta)=\sum_{n=1}^{\infty}B_n\, r^n \sin n\theta \text{------}\rightarrow (5)$$

<u>Apply 4th Boundary Condition in (5):</u>

$$u(a,\theta)=\sum_{n=1}^{\infty}B_n\, a^n \sin n\theta = f(\theta)=100^{\circ}c$$

To find B_n: Expand in fourier half range sine series

$$B_n\ a^n=\frac{2}{\pi}\int_0^{\pi}100\sin\left(n\pi\right)\, d\theta$$

$$=\frac{200}{\pi}\left[\frac{-\cos^{-1} n\theta}{n}\right]_0^{\pi}$$

$$=\frac{-200}{n\pi}\left[\cos n\pi-\cos 0\right]$$

$$B_n\ a^n=\frac{200}{n\pi}\left(1-\left(-1\right)^n\right)$$

$$B_n\ a^n=\begin{cases}\dfrac{400}{n\pi}, & n\ \ is\ \ odd\\ 0, & n\ \ is\ \ even\end{cases}$$

$$B_n=\begin{cases}\dfrac{400}{n\pi\ a^n}, & n\ \ is\ \ odd\\ 0, & n\ \ is\ \ even\end{cases}$$

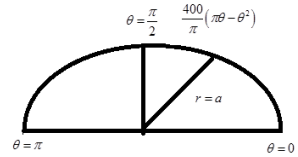
Substitute in eqn (5)

$$u(r,\theta)=\sum_{n=1}^{\infty}\frac{400}{n\pi\ a^n}r^n \sin n\theta$$

2. **A semicircular Plate of radius 10 cm has insulated faces and heat flows in tankers. The bounding diameter is kept at 0°C and on the circumference and on the surfaces the temp is given by u(10, θ)=** $\frac{400}{\pi}\left(\pi\theta-\theta^2\right),\ 0<\theta<\pi$ **. Find the Steady State temp distribution.**

<u>Solution:</u>

The 2 dimensional heat equation in Polar Co-ordinates is $r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0$



The boundary conditions are,

1. u(r, 0) = 0, $\quad 0 < r < a$
2. u(r, π) = 0, $\quad 0 < r < a$
3. $r \to 0$, $\quad u \to 0$, $\quad 0 < r < a$
4. $u(10, \theta) = f(\theta) = \dfrac{400}{\pi}\left(\pi\theta - \theta^2\right)$,

The suitable solution which satisfy our boundary conditions are

$$u(r,\theta) = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta) ------ \to (1)$$

<u>Apply 1st Boundary Condition in (1) :</u>

$$u(r,0) = (c_1 r^p + c_2 r^{-p})\left[c_3 \cos(0) + c_4 \sin(0)\right]$$

$$u(r,0) = (c_1 r^p + c_2 r^{-p})c_3 = 0$$

But $(c_1 r^p + c_2 r^{-p}) \neq 0$

$$\therefore c_3 = 0$$

Substitute in equation (1)

$$u(r,\theta) = (c_1 r^p + c_2 r^{-p})c_4 \sin p\theta ------ \to (2)$$

<u>Apply 2nd Boundary Condition in (2):</u>

$$u(r,\pi) = (c_1 r^p + c_2 r^{-p})c_4 \sin p\pi = 0$$

But $(c_1 r^p + c_2 r^{-p}) \neq 0 \quad and \quad c_3 \neq 0$

$$\therefore \ \sin p\pi = 0$$

$$p\pi = \sin^{-1}(0)$$

$$p\pi = n\pi$$

$$\boxed{p = n}$$

Substitute in equation (1)

$$u(r,\theta) = (c_1 r^n + c_2 r^{-n})c_4 \sin n\theta ------ \to (3)$$

Apply 3rd Boundary Condition in (3):

When $r \to 0 \quad \Rightarrow \quad c_2 \to 0$

Substitute c_2 value in eqn (3)

$$u(r,\theta) = c_1 r^n c_4 \sin n\theta$$

$$u(r,\theta) = B_n\, r^n \sin n\theta \text{------} \to (4)$$

The most general solution of eqn (4) is

$$u(r,\theta) = \sum_{n=1}^{\infty} B_n\, r^n \sin n\theta \text{------} \to (5)$$

Apply 4th Boundary Condition in (5):

$$u(10,\theta) = \sum_{n=1}^{\infty} B_n 10^n \sin n\theta = f(\theta) = \frac{400}{\pi}\left(\pi\theta - \theta^2\right)$$

To find B_n: Expand in fourier half range sine series

$$B_n\, 10^n = \frac{2}{\pi}\int_0^{\pi} \frac{400}{\pi}\left(\pi\theta - \theta^2\right)\sin\left(n\theta\right)\, d\theta$$

$$u = \pi\theta - \theta^2 \qquad v = \sin n\theta$$

$$u' = \pi - 2\theta \qquad v_1 = \frac{-\cos n\theta}{n}$$

$$u'' = -2 \qquad v_2 = \frac{-\sin n\theta}{n^2}$$

$$u''' = 0 \qquad v_3 = \frac{\cos n\theta}{n^3}$$

$$= \frac{800}{\pi^2}\left[\left(\pi\theta - \theta^2\right)\left(\frac{-\cos n\theta}{n}\right) - \left(\pi - 2\theta\right)\left(\frac{-\sin n\theta}{n^2}\right) + \left(-2\right)\left(\frac{\cos n\theta}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{800}{\pi^2}\left[\frac{-2\cos n\pi}{n^3} + \frac{2\cos 0}{n^3}\right]$$

$$B_n\, 10^n = \frac{800 \times 2}{n^3\pi^2}\left(-\left(-1\right)^n + 1\right)$$

$$B_n\, 10^n = \frac{1600}{n^3\pi^2}\left(1 - \left(-1\right)^n\right)$$

$$B_n\ 10^n = \begin{cases} \dfrac{3200}{n^3\pi^2}, & n\ \ is\ \ odd \\ 0, & n\ \ is\ \ even \end{cases}$$

$$B_n = \begin{cases} \dfrac{3200}{n^3\pi^2\ 10^n}, & n\ \ is\ \ odd \\ 0, & n\ \ is\ \ even \end{cases}$$

Substitute in eqn (5)

$$u(r,\theta) = \sum_{n=1}^{\infty} \frac{3200}{n^3\pi^2\ 10^n} r^n \sin n\theta$$

UNIT-IV

APPLIED STATISTICS.

Curve fitting by the method of least square:

1. Straight line,y=ax+b(or)y=a+bx
2. Parabola,$y=ax^2+bx+c$ (or) $y=a+bx+cx^2$
3. Exponential, $y=ae^{bx}$

Straight line:

St. line equation	Normal equation
1. y=ax+b	$\sum xy=a\sum x^2+b\sum x$ $\sum y=a\sum x+nb$
2. y=a+bx	$\sum xy=a\sum x+b\sum x^2$ $\sum y=na+b\sum x$

1. **Fit a st. line for the following independent variable by the method of least squaring.**

x:	**1**	**2**	**3**	**4**	**5**
y:	**4**	**3**	**6**	**7**	**11**

Solution:

Let the st. line eqn be, y =ax+b ----------(1)

The normal equation is,

$\sum xy = a\sum x^2 + b\sum x$ ----------(2)

$\sum y = a\sum x + nb$ ----------(3)

Hence, n=5

x	y	x^2	xy
1	4	1	4
2	3	4	6
3	6	9	18
4	7	16	28
5	11	25	55
$\sum x=15$	$\sum y=31$	$\sum x^2 = 55$	$\sum xy=111$

Sub these values in eqn (2) and (3)

111=55a +15b

31=15a+15b----------(2)

Solving this eqn we get

a =1.8 and b =0.8

Substitute in y=ax+b we get

y =1.8x+0.8

2.Fit a 1st degree curve to the following data ad eliminate the value of y when x=73.

x :	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**
y :	**1**	**3**	**5**	**10**	**6**	**4**	**2**	**1**

Solution:

The St. line equation is y=ax+b ----------(1)

The normal Equations are,

$\sum xy = a\sum x^2 + b\sum x$ ----------(2)

$\sum y = a\sum x + nb$ ----------(3)

Hence, n=8

x	y	x^2	xy
10	1	100	10
20	3	400	60

30	5	900	150
40	10	1600	400
50	6	2500	300
60	4	3600	240
70	2	4900	140
80	1	6400	80
$\sum x=360$	$\sum y=32$	$\sum x^2=20400$	$\sum xy=1380$

Substitute these values in equation (1) and (2) we get

$$1380=20400a+360b$$

$$32=360a+8b$$

Solving these eqn we get

a =-0.014 and b=4.642

Substitute in y=ax+b

y =-0.014x+4.642

Given When x=73, sub in above equation

y =-0.014(73)+4.642

y =3.61

3.Fit a st. line for the curve y=ax+b/x for the following data:

x :	**1**	**2**	**3**	**4**
y :	**- 1.5**	**0.99**	**3.88**	**7.66**

Solution:

The given st. line equation is,

$y = ax+\frac{b}{x}$ ----------(*)

$xy= ax^2+b$

let, Y =xy

$X = x^2$

Sub in eqn (*)

Y =aX+b ----------(1)

The normal equations are,

$\sum XY = a\sum X^2 + b\sum X$----------(2)

$\sum Y = a\sum X + nb$----------(3)

x	y	$X=x^2$	Y=xy	X^2	XY
1	-1.5	1	-1.5	1	-1.5
2	0.99	4	1.98	16	7.92
3	3.88	9	11.64	81	104.76
4	7.66	16	30.64	256	490.24
$\sum X = 10$	$\sum Y = 6.16$	$\sum X = 30$	$\sum Y = 42.76$	$\sum X^2 = 354$	$\sum XY = 601.42$

Sub in eqn (1) and (2) we get

$$601.42 = 354a + 30b$$

$$\underline{42.76 = 30a + 4b}$$

$$a = 2.17$$

$$b = -5.63$$

$$Y = 2.17X - 5.63$$

Replace X and Y values here

$$Xy = 2.17\,x^2 - 5.63$$

$$y = 2.17x - 5.63/x$$

Fitting a Parabola:

Parabola Equation	Normal Equation
1.$y=ax^2+bx+c$	$\sum x^2y = a\sum x^4 + b\sum x^3 + c\sum x^2$ $\sum xy = a\sum x^3 + b\sum x^2 + c\sum x$ $\sum y = a\sum x^2 + b\sum x + nc$
2.$y=a+bx+cx^2$	$\sum x^2y = a\sum x^2 + b\sum x^3 + \sum x^4$ $\sum xy = a\sum x + b\sum x^2 + c\sum x^3$ $\sum y = na + b\sum x + c\sum x^2$

1. **Fitting a parabola by the method of least square.**

x :	**1**	**2**	**3**	**4**	**5**
y:	**2**	**3**	**5**	**8**	**10**

Solution:

Let, the parabola equation be,

$y = ax^2+bx+c$ ----------(1)

The normal equations are,

$\sum x^2y=a\sum x^4+b\sum x^3+c\sum x^2$ ----------(2)

$\sum xy=a\sum x^3+b\sum x^2+c\sum x$ ----------(3)

$\sum y=a\sum x^2+b\sum x+nc$ ----------(4)

x	**y**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
1	2	1	1	1	2	2
2	3	4	8	16	6	12
3	5	9	27	81	15	45
4	8	16	64	256	32	128
5	10	25	125	625	50	250
$\sum x = 15$	**$\sum y = 28$**	**$\sum x^2 = 55$**	**$\sum x^3 = 225$**	**$\sum x^4 = 979$**	**$\sum xy = 105$**	**$\sum x^2y = 437$**

Substitute in eqn (1),(2) and (3)

$$437=979a + 225b + 55c$$

$$105=225a + 55b + 15c$$

$$28=55a + 15b + 5c$$

Solving this we get

$a = 0.2142$, $b = 0.8142$, $c = 0.8$

Sub a,b,c values in $y = ax^2+bx+c$

$$y = 0.2142x^2 + 0.8142x + 0.8$$

2.Fit a parabola of the curve $y=a+bx+cx^2$ by the method of least square.

x :	**2**	**4**	**6**	**8**	**10**
y :	**3.07**	**12.85**	**31.47**	**57.38**	**91.29**

Solution:

The equations of the parabola is,

$y = a+bx+cx^2$----------(1)

The normal equations are,

$\sum x^2y = a\sum x^2 + b\sum x^3 + \sum x^4$ ----------(2)

$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$ ----------(3)

$\sum y = na + b\sum x + c\sum x^2$ ----------(4)

x	**y**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
2	3.07	4	8	16	6.14	12.28
4	12.85	16	64	256	51.4	205.6
6	31.47	36	216	1296	188.82	1132.92
8	57.38	64	512	4096	459.04	3672.32
10	91.29	100	1000	100000	912.9	9129
$\sum x = 30$	$\sum y =$ 196.06	$\sum x^2 = 220$	$\sum x^3 =$ 1800	$\sum x^4 =$ 15664	$\sum xy =$ 1618.3	$\sum x^2y =$ 14152.12

Sub in eqn (2), (3), (4) we get

$$14152.112 = 220a + 1800b + 15664c$$

$$1618.3 = 30a + 220b + 1800c$$

$$196.06 = 5a + 30b + 220c$$

$$a = 0.696$$

$$b = -0.8550$$

$$c = 0.9919$$

sub in eqn (1)

$$y = 0.696 - 0.855x + 0.9919\,x^2$$

Fitting of exponential curve:

Type-I:

Let, $\mathbf{y = ae^{bx}}$

1.Find the curve for $y = ae^{bx}$ for the following data:

x :	**0**	**2**	**4**	**6**	**8**
y :	**150**	**63**	**28**	**12**	**5.6**

Solution:

Let $y = ae^{bx}$----------(1)

Taking log on both sides,

$\log y = \log (ae^{bx})$

$= \log a + \log e^{bx}$

$\log y = \log a + b x \log e$

$Y = A + B x$, ----------(*) which is a straight line equation

Where $Y = \log y$

$A = \log a$

$B = b \log e$

The normal equations are,

$\sum xY = A \sum x + B \sum x^2$ ----------(2)

$\sum y = n A + B \sum x$----------(3)

x	Y	x^2	Y=logy	xY
0	150	0	2.1760	0
2	63	4	1.7993	3.5986
4	28	16	1.4471	5.7886
6	12	36	1.0791	6.4750
8	5.6	64	0.7481	5.9855
$\sum x = 20$		$\sum x^2 = 120$	$\sum Y = 7.2496$	$\sum xY = 21.8477$

Sub these values in eqn (2) and (3) we get

$21.8477 = 20 A + 120B$

$7.2496 = 5A + 20B$

$A=2.1649$

$B=-0.1786$

Sub in (*) we get

$Y = 2.1649X - 0.1786$

But

$A = \log a$

$a = \text{antilog } A$

$a = 146.1840$

$B = b \log e$

$b = B/\log e$

$=-0.1786/0.4343$

$b = -0.4112$

sub in (1)

$y=(146.1840)\ e^{-0.4112x}$

2.Find the curve $y=ae^{bx}$ for the following data:

x :	**0**	**5**	**8**	**12**	**20**
y :	**3.0**	**1.5**	**1.0**	**0.55**	**0.18**

Given:

$y = ae^{bx}$

Taking log on both sides,

$\log y = \log (ae^{bx})$

$= \log a + \log e^{bx}$

$\log y = \log a + b \times \log e$

$Y = A + B x$, ----------(1) which is a straight line equation

Where $Y = \log y$

$A = \log a$

$B = b \log e$

The normal equations are,

$\sum xY = A \sum x + B \sum x^2$----------(2)

$\sum Y = n A + B \sum x$----------(3)

x	y	Y=log y	x^2	xY
0	3	0.4771	0	0
5	1.5	0.1760	25	0.8804
8	1	0	64	0
12	0.55	-0.2596	144	-3.1156
20	0.18	-0.7447	400	-14.8945
$\sum x = 45$		$\sum Y = -0.3512$	$\sum x^2 = 633$	$\sum xY = -17.1297$

Sub in eqn (2) and (3)

$-17.1297 = 45\,A + 633\,B$

$-0.3512 = 5\,A + 45\,B$

$A=0.4811$

$B=-0.0612$

$Y=0.4811 - 0.0612x$

But $a = \text{antilog } A$

$a = 3.0276$

$b= B/\log e$

$= -0.0612/0.4343$

$b = -0.1409$

sub in eqn (1)

$y=(3.0276)e^{-0.1409x}$

Type – II:

Let, $y=ab^x$

1.Fit a curve of the form $y=a.b^x$ from the following table:

x :	**0**	**1**	**2**	**3**	**4**	**5**	**6**
y :	**32**	**47**	**65**	**92**	**132**	**190**	**275**

Solution:

Let, $y=ab^x$ ----------(1)

Taking log on both sides,

$\log y = \log [ab^x]$

$\log y= \log a + x \log b$

$Y = A + B\,x$ ----------(2)

Where $Y=\log y$; $A=\log a$; $B=\log b$

$Y=A + B\,x$, which is a straight line equation

The normal equations are,

$\sum xY= A \sum x + B \sum x^2$----------(3)

$\sum Y = n A + B \sum x$ ----------(4)

x	y	Y=log y	x^2	xY
0	32	1.5051	0	0
1	47	1.6720	1	1.6720
2	65	1.8129	4	3.6258
3	92	1.9637	9	5.8913
4	132	2.1205	16	8.4822
5	190	2.2787	25	11.3937
6	275	2.4393	36	14.6359
$\sum x = 21$		$\sum Y = 13.7922$	$\sum x^2 = 91$	$\sum xY = 45.7009$

Sub in eqn (3) and (4) we get

$45.7009 = 21 A + 91 B$

$13.7922 = 7 A + 21 B$

$A = 1.5069$

$B = 0.1544$

Sub in eqn (2)

$Y = 1.5069 + 0.1544 x$

But $a = \text{antilog } A$

$a = 32.1292$

$b = \text{antilog } B$

$b = 1.4269$

Sub in eqn (1)

$y = 32.1292 (1.4269)^x$

Type – III :

Let , $y = a x^b$ be the power curve.

1.Fit the power curve of the form $y = a x^b$ for the following data:

x :	**1**	**2**	**3**	**4**	**5**
y :	**7.1**	**27.8**	**62.1**	**110**	**161**

Solution:

$y = a x^b$

Taking log on both sides,

$$\log y = \log a + b \log x$$

$$Y = A + bX \text{ (Straight line equation)}$$

The normal equations are,

$$\sum XY = A \sum X + b \sum X^2 \text{ ----------(1)}$$

$$\sum Y = n A + b \sum X \text{ ----------(2)}$$

x	y	X=log x	Y=log y	X^2	XY
1	7.1	0	0.8512	0	0
2	27.8	0.3010	1.440	0.0906	0.4346
3	62.1	0.4771	1.7930	0.2276	0.8554
4	110	0.6020	2.0413	0.3624	1.2289
5	161	0.6989	2.2068	0.4885	1.5423
$\sum X = 15$		$\sum X = 2.079$	$\sum Y = 8.3363$	$\sum X^2 = 1.1691$	$\sum XY = 4.0612$

Substitute in eqn (1) and (2)

$$4.0612 = 2.079 A + 1.1691 b$$

$$8.3363 = 5 A + 2.079 b$$

Solving this we get

$$A=0.8552$$

$$b =1.9529$$

$$Y = 0.8552 + 1.9529 x$$

$$a = \text{anti log } (A) = 7.1647$$

Sub a and b values in (1)

$$Y = 7.1647(x)^{1.9529}$$

Sampling Theory:

1.If $n<30$ is called small sample.

2.If $n>30$ is called large sample.

Statistics and parameters:

μ, σ = mean and standard deviation of population.

x, s = mean and standard deviation of sample.

σ^2= Variance of population.

Level of significance		
Critical value		
1. Two tailed test	1.96	2.58
2.Right tailed test	1.645	2.33
3.Left tailed test	1.645	2.33

If the size of the sample n>30. Then it is called large sample.

Five types of large samples:

i) Test of significance of single proportion.

ii) Test of significance of different proportion.

iii) Test of significance of single mean.

iv) Test of significance of difference of mean.

v) Test of significance of difference standard deviation

Type 1 - Test of significance of single proportion.

PROCEDURE:

1. P, p, n,

p = X / n = sample proportion

P= Population proportion

n = sample size

2. Assume null hypothesis, H_0

   Alternative hypotheis,H_1

3. Calculate Z value

   Z=(p – P) / $\sqrt{PQ/n}$

   Q=1 – P.

4. Table value at 5% (or) 1% level of significance.

5. Calculte value less than table value, null hypothesis H_0 is accepted.

6. Calculate value greater than table value, null hypothesis H_0 is rejected.

PROBLEMS:

1. The manufactures claimed that at least 95% of equipment's which he supplied to a factory conformed to the specifications. An examination of sample of 200 pieces of equipment's revealed at 18 were faulty, test his claim at 5% level of significance.

Solution:

Step 1:

Here, n=200

X = no of pieces containing to speciation's in the sample

X = 200 – 18 = 182

P= 95% = 0.95

Q= 1 – P = 0.05

p =sample proportion conformed to specification

p = X / n = (200 – 18)/200

p= 0.91

Step 2:

Assume null hypothesis H_0 :

P=0.95

Alternative hypothesis, H_1 :

P ≠ 0.95 [Two tailed test]

Step 3: Test statistic

∴ Z=(p – P)/$\sqrt{PQ/n}$

=(0.91 – 0.95) / $\sqrt{(0.95 * 0.05)/200}$

= -2.5955

|Z| = 2.5955

Step 4:

∴Table value for 5% level of significance is z =1.96

∴ Calculate value is greater than table value i.e. 2.59>1.96

∴H_0 is rejected.

Conclusion:

Hence the manufacture claim is rejected.

2.In a sample of 1000 people in Karnataka, 540 are rice eaters and the rest are wheat eaters, can we assume that both rice and wheat eaters are equally popular. In this state at 1% level of significance.

Solution:

Step 1:

Given n=1000

X = 540

P=50%=0.5

Q = 1 – P = 1 – ½ = ½ = 0.5

p =X/n=540/1000=0.54

Step 2:

Assume null hypothesis, H_0

P=0.5

Alternate hypothesis, H_1:

P≠0.5 [Two tailed test]

Step 3: Test statistic

Z=(p – P) / $\sqrt{PQ/n}$

=(0.54-0.5)/$\sqrt{(0.5 * 0.5)/1000}$

=2.5298.

Step 4:

∴ Table value for 1% level of significance, Z=2.58.

∴ Calculate value is lesser than table value i.e 2.52<2.58.

∴$H_0$is accepted.

Conclusion:

We may conclude that both rice and wheat eaters are equally popular in this state.

3. In a big city, 325 men out of 600 men were found to be smokers. Does this information support the conclusion that majority of the men in this city are smokers.

Solution:

Step 1:

Given n= 600

X = 325

P=50%=0.5

Q=1 – P=0.5

p = X/n= 325/600= 0.5416

Step 2:

Assume null hypothesis, H_0

P=0.5

Assume Alternate hypothesis, H_1:

P ≠0.5 [two tailed test]

Step 3: Test statistic:

Z=(p – P)/$\sqrt{PQ/n}$

=(0.5416 – 0.5)/$\sqrt{(0.5 * 0.5)/600}$

=2.0379

Step 4:

Table value for 5% level of significance,Z=1.96

∴ Calculate value isgreater than table value

∴ 2.04>1.96

i.e. $H_0$is rejected.

Conclusion:

∴ Majority of men are smokers in the city.

4. In a factory sample 400 parts are manufactured, the no. of deflective parts was found to be 30, the company however claims that only 5% of that product is deflective.

Solution:

Step 1:

Given n=400

X = 30

p =X/n=30/400=0.075

P=population proportion=5%=0.05

Q=1 – P=0.95

Step 2:

Assume null hypothesis H_0 :

P=0.05

Assume alternate hypothesis H_1:

P ≠0.05 [two tailed test]

Step 3Test statistic:

Z=(p – P)/$\sqrt{PQ/n}$

=(0.075 – 0.05)/$\sqrt{(0.05 * 0.95)/400}$

= 2.2941

Step 4:

∴Table value for 5% level of significance is |Z|=1.96

∴Calculate value is greater than the tabulated value.

∴2.2941>1.96

i.e. H_0 is rejected.

Conclusion:

∴The company claims that 5% of sample is not acceptable.

5. A die was thrown 9000 times and a throw is observed 3220 times. Show that the die cannot be regarded as an unbiased.

Solution:

Step 1:

Given n =9000

p =X/n=3220/9000=0.3578

P=Population proportion of success

=(1/6)+(1/6)=2/6=1/3=0.3333

Q=1 – P=0.6667.

Step 2:

Assume null hypothesis H_0 :

P=0.3333

Assume alternate hypothesis H_1:

P≠0.3333[two tailed test]

Step 3: Test statistic:

Z=(p – P)/$\sqrt{PQ/n}$

=(0.3578 – 0.333)/$\sqrt{(0.3333 * 0.6667)/9000}$

=4.9306.

Step 4:

The table value for 5% level of significance is |Z|=1.96.

∴Calculate value is greater than the tabulated value.

∴4.9306>1.96

i.e. $H_0$is rejected.

Conclusion:

Therefore the dice is biased.

<u>NOTE:</u>

1. 98% of confident limit,

   P=p $\pm$ 2.33$\sqrt{pq/n}$

2. 99% of confident limit,

   P=p $\pm$2.58$\sqrt{pq/n}$

3. 95% of confident limit,

   P=p $\pm$1.96$\sqrt{pq/n}$

4. Limits for population P=p $\pm$3$\sqrt{pq/n}$

1. A random sample of 500 apples was taken from a large container and 60 were found to be bad. Obtain 98% of confident limit for the number of bad apples in the container.

Solution:

Step 1:

Given n=500

X = 60

p =60/500=0.12

q=1 – p=0.88

Step 2:

w.k.t.

98% of confident limit for population proportion is given by,

$P=p\pm 2.33\sqrt{pq/n}$

$=0.12\pm\ 2.33\sqrt{(0.12*0.88)/500}$

$=0.12\pm\ 0.0388$

P =0.1538 ; P=0.0862

Conclusion:

∴ 98% of confident limit for the no. of bad appler in the container are 0.1538,0.0862.

2. A random sample of 500 pineapples was taken from a large container and 65 were found to be bad. Find the percentage of bad pineapples in the container.

Solution:

Step 1:

Given n=500

X = 65

p=65/500=0.13

q=1 – p=0.87

Step 2:

Limits for population proportion is given by

$P=p\pm 3\sqrt{pq/n}$

$=0.13\pm 3\sqrt{(0.13*0.87)/500}$

$P=0.13\pm 0.0451$

P=0.1751 ; P=0.0849

Conclusion:

∴ The percentage of bad pineapples in the container are 0.1751 , 0.0849.

Type 2 - Test of significance of different proportion:

1. If sample proportions $p_{1,}$ p_2 are given then $z=\dfrac{|p_1-p_2|}{\sqrt{PQ\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$

To find P:

$$P=\frac{n_1p_1+n_2p_2}{n_1+n_2}$$

To find Q:

Q=1 – P

2. If the sample proportions are not given then $z=\dfrac{|p_1-p_2|}{\sqrt{\left(\dfrac{P_1Q_1}{n_1}+\dfrac{P_2Q_2}{n_2}\right)}}$
3. If difference is given in proportion, then $z=\dfrac{|(p_1-p_2)-(P_1-P_2)|}{\sqrt{PQ\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$
4. Test the signi difference b/w both sample and population proportion

$$z=\frac{|p_1-P_1|}{\sqrt{\left(\dfrac{n_2PQ}{n_1(n_1+n_2)}\right)}}$$

1. A random sample of 400 men and 600 women were asked whether they would like to have a school near this residency. 200 men and 325 women were in favor of their proposal. Test the hypothesis that proportion of men and women in favor of the proposal are same at 5% level.

SOLUTION:

Step 1:

n_1 =400	n_2 =600
$X_1 = 200$	$X_2 = 325$
p_1 =200/400	p_2 =325/600
p_1 =0.5	p_2 =0.5417
q_1 =1 – P_1 =0.5	q_2 =1 – P_2 = 0.4583

Step 2:

Null hypothesis, H_0:

$$p_1 = p_2$$

Alternate hypothesis,H_1 :

$$p_1 \neq p_2 \text{ (Two tailed test)}$$

Step 3: Test Statistic

$$z = \frac{|p_1 - p_2|}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

=((400*0.5)+(600*0.541))/(400+600)

=0.5250

Q=1 – P=0.4749

$$Z=|0.5 - 0.54171| / \sqrt{(0.5250*0.4749)\left(\frac{1}{400} + \frac{1}{600}\right)}$$

Z = 1.2937

Step 4:

At 5% level of significance, Z=1.96

∴calculated value<Tabulated value

i.eH_0 is accepted.

Conclusion:

∴ The proportion of men and women in favour of proposals are same.

2. In a two large population there are 30% and 25% respectively of fair haired people. In this difference likely to be hidden in a sample of 1200 and 900 respectively from the two population.

SOLUTION:

Step 1:

$n_1 = 1200$	$n_2 = 900$
$P_1 = 30\% = 0.3$	$P_2 = 30\% = 0.3$
$Q_1 = 1 - P_1 = 0.7$	$Q_2 = 1 - P_2 = 0.75$

Step 2:

Assume null hypothesis, H_0:P_1=P_2

Assume Alternate hypothesis,H_1: $P_1 \neq P_2$ [two tailed test]

Step 3: Test Statistic

$$z = \frac{|p_1 - p_2|}{\sqrt{\left(\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}\right)}}$$

$$=(0.3-0.25)/\sqrt{\left(\frac{0.3*0.7}{1200}\right)+\left(\frac{0.25*0.75}{900}\right)}$$

=2.5537

Step 4:

At 5% level of significance, Z=1.96.

The calculated value>tabulated value

Conclusion:

i.eH_0 is rejected.

3. The manufacturing form claim that its brand A product out sells the brand B product by 8%, it found that 42 out of sample of 200 persons prefer brand A and 18 out of another sample of 100 persons prefer brand B. Test whether 8% difference is valid claim.

SOLUTION:

$n_1 = 200$ $\quad$ $n_2 = 100$

$X_1 = 42$ $\quad$ $X_2 = 18$

$p_1 = 42/200$ $\quad$ $p_2 = 18/100$

$p_1 = 0.21$ $\quad$ $p_2 = 0.18$

$q_1 = 1 - P_1 = 0.79$ $\quad$ $q_2 = 1 - P_2 = 0.82$

The difference between population proportion is $P_1 - P_2$ =8%=0.08.

Step 2:

Assume Null hypothesis, H_0:P_1-P_2=0.08

Assume alternate hypothesis,H_1: P_1-P_2≠0.08

Step 3: Test Statistic

$$z = \frac{|(p_1 - p_2) - (P_1 - P_2)|}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

P=((0.21*200)+(0.18*100)) / (200+100)

P=0.2

Q=1 – P=0.8

$$z = \frac{|(0.21 - 0.18) - (0.08)|}{\sqrt{(0.2*0.8)\left(\frac{1}{200} + \frac{1}{100}\right)}}$$

=0.05/0.0489

Z=1.0206.

Step 4:

At 5% level of significance, Z=1.96.

∴ The tabulated value is greater than th calculated value

i.e. H_0 is accepted.

Conclusion:

∴ The 8% difference is valid claim.

4. In a random sample of 400 students of university teaching department, it was found that 300 students failed in the examination. In another random sample of 500 students of the affiliated college, the no. of failures in the same examination was found to be 300. Find out whether the proportion of failures in the university teaching department is significantly greater than the proportion of failures in the university teaching department and affiliated colleges taken together.

SOLUTION:

Step 1:

$n_1 = 400$ $\quad$ $n_2 = 500$

$X_1 = 300$ $\quad$ $X_2 = 300$

$p_1 = 300/400$ $\quad$ $p_2 = 300/500$

$p_1 = 0.75$ $\quad$ $p_2 = 0.6$

$q_1 = 1 - P_1 = 0.25$ $\quad$ $q_2 = 1 - P_2 = 0.4$

p_1 = population proportion of university teaching department.

P_1 = Population proportion of both university teaching department and affiliated colleges

Step 2:

∴ Assume Null hypothesis, H_0: $P_1 = P$

Alternate hypothesis, H_1: $P_1 \neq P$ [two tailed test].

Step 3: Test Statistic

$$z = \frac{|p_1 - P_1|}{\sqrt{\left(\frac{n_2 PQ}{n_1(n_1+n_2)}\right)}}$$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{(400*0.75)+(500*0.6)}{400+500}$$

=0.6667

Q= 1 – P=0.3333

$$z = \frac{0.75 - 0.6667}{\sqrt{\frac{500}{400(900)}*0.6667*0.3333}}$$

Z=4.7416

Step 4:

At 5% level of significance, Z=1.96

∴ Calculated value is greater than the tabulated value.

i.e.H_0 is rejected.

Conclusion:

∴ We conclude that there is significance difference between P_1 and P.

5. In a large city A, 20% of random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of random sample of 1600 school boys had the same defect. Is there any difference between the proportions of significance?

SOLUTION:

Step 1:

$n_1 = 900$ $\qquad$ $n_2 = 1600$

$p_1 = 20\% = 0.2$ $\qquad$ $p_2 = 18.5\% = 0.185$

$q_1 = 1 - P_1 = 0.8$ $\qquad$ $q_2 = 1 - P_2 = 0.815$

Step 2:

Null hypothesis, H_0: $p_1 = p_2$

Alternate hypothesis, H_1 : $p_1 \neq p_2$ [two tailed test]

Step 3: Test Statistic

$$z = \frac{|p_1 - p_2|}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{(900*0.2) + (1600*0.185)}{900 + 1600}$$

P=0.1904

Q=0.8096

$$z = \frac{|0.2 - 0.185|}{\sqrt{0.1904*0.8096(\frac{1}{900} + \frac{1}{1600})}}$$

z =0.9169

Step 4:

At 5% level of significance, the tabulated value for Z=1.96.

Here, tabulated value is greater than calculated value.

$\therefore$ H_0 is accepted.

Conclusion:

Thus there is a difference between the proportion of significance.

TYPE 3 - TEST OF SIGNIFICANCE OF SINGLE MEAN:

To test whether the difference between population mean and sample mean is significant (or) not,

i) If σ is given, then $z = \frac{|\bar{x} - \mu|}{\sigma / \sqrt{n}}$

ii) If σ is not given, then $z = \frac{|\bar{x} - \mu|}{s / \sqrt{n}}$

Where $\bar{x}$ =sample mean

μ = population mean

σ= population standard deviation

S=sample standard deviation

n = sample size

CONFIDENT LIMIT (OR) FIDUCIAL LIMIT:

1. 95% of confident limit, $\mu = \bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$
2. 98% of confident limit, $\mu = \bar{x} \pm 2.33\frac{\sigma}{\sqrt{n}}$
3. 99% of confident imit,$\mu = \bar{x} \pm 2.58\frac{\sigma}{\sqrt{n}}$

1. **A sample of 900 members has a mean of 3.4cm and S.D. is 2.61cm. If the sample from a large population of mean 3.25cm and S.D is 2.61cm. If the population is normal and its mean is unknown, find the 95% of fiducial limit of the true mean.**

SOLUTION:

Step 1:

n =900

$\bar{x}$ =3.4cm

s =2.61cm

μ =3.25cm

σ =2.61cm

Step 2:

Assume null hypothesis, H_0:

μ =3.25

Assume alternate hypothesis,H_1 :

$\mu \neq 3.25$

Step 3: Test Statistic

Here, σ is given then $z = \frac{|\bar{x} - \mu|}{\sigma / \sqrt{n}}$

$$z = \frac{|3.4 - 3.25|}{2.61/\sqrt{900}}$$

Z=1.7241

Step 4:

At 5% level of significance, the tabulated value of Z is 1.96.
The calculated value <tabulated value .
i.e. ,$H_0$is accepted.

Conclusion:

The population mean is significant

At 95% of confident limit are

$\mu = \bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

$=3.4 \pm 1.96(2.61/\sqrt{900})$.

$=3.4 \pm 0.1705$

$\mu = 3.5705$;$\mu=3.2295$

2. **A normal population has a mean of 6.48 and a S.D of 1.5 In a sample of 400 members, mean is 6.75. Is the difference significant?**

Solution:

Step 1:

GIVEN:

n =400

μ=6.48

σ=1.5

$\bar{x}$ =6.75

Step 2:

Assume null hypothesis,H_0:

μ =6.48

Assume alternate hypothesis,H_1 :

$\mu \neq 6.48$

Step 3: Test Statistic

Here, σ is given then $z = \frac{|\bar{x} - \mu|}{\sigma/\sqrt{n}}$

$$z = \frac{|6.75 - 6.48|}{\frac{1.5}{\sqrt{400}}}$$

z =3.6.

Step 4:

At 5% level of significance, the tabulated value of Z is 1.96.

∴ Tabulated value < calculated value.

i.e. H_0 is rejected.

Conclusion:

∴ The difference is significant.

3. **An insurance agent has claimed that the average age of policy holders who issue through him is less than the average for all agents which is 30.5 years. A random sample of 100 policy holders who had issued through him gave the following age distribution.**

Age:	**16-20**	**21 – 25**	**26 – 30**	**31 – 35**	**36 – 40**
No.of persons:	**12**	**22**	**20**	**30**	**16**

Calculate the arithmetic mean and S.D. of this distribution and use these values to test his claim at 5% level of significance.

SOLUTION:

Step 1:

n =100

μ=30.5

A=middle value of x_i = 28

h =common difference=5

Age	No. of persons (f)	Mid pt. x_i	d $=\frac{x_i-A}{h}$	f_d	f_{d^2}
16 – 20	12	18	-2	-24	48

21 – 25	22	23	-1	-22	22
26 – 30	20	**28 = A**	0	0	0
31 – 35	30	33	1	30	30
36 – 40	16	38	2	32	64
	100			16	164

Arithmetic mean, $\bar{x} = A + \frac{h\sum f_d}{\sum f}$

=28+(5(16)/100)

=28.8

Standard deviation, $s = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$

$$= 5\sqrt{\frac{164}{100} + \left(\frac{16}{100}\right)^2}\, h$$

s = 6.3529

Step 2:

Assume null hypothesis, H_0:

μ =30.5

Assume alternate hypothesis, H_1 :

$\mu \neq 30.5$

Step 3: Test Statistic

$$z = \frac{|\bar{x} - \mu|}{s/\sqrt{n}}$$

$$z = \frac{|28.8 - 30.5|}{\frac{6.3529}{\sqrt{100}}}$$

=2.6759

Step 4:

At 5% level of significance the tabulated value of Z=1.96

∴ Calculatedvalue >Tabulated value.

i.e. $H_0$is rejected.

Conclusion:

His claim is rejected.

TYPE – 4

TEST OF SIGNIFICANCE OF DIFFERENCE OF MEAN

To test whether the significant difference between $\bar{x}_1$ and $\bar{x}_2$

1. If the population S.D is given (i.e) σ is given then,

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

2. If $\sigma_1 = \sigma_2 = \sigma$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

3. If the population standard deviation is not given (i.e) σ is not given

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

PROBLEMS

1. **The mean of two large sample of size 2000 and 1000 of the mean are 68.0 and 67.5 respectively can the sample mean regarded as drawn from the same population standard deviation of 2.25.**
Solution:

Step 1:

$$\bar{x}_1 = 68$$
$$\bar{x}_2 = 67.5$$
$$n_1 = 2000$$
$$n_2 = 1000$$
$$\sigma = 2.25$$

Step 2:

Null hypothesis $H_0 \ \mu_1 = \mu_2$

Alternate hypothesis $H_1 \ \mu_1 \neq \mu_2$ (two tailed test)

Step 3: Test statistics

Here $\sigma_1 = \sigma_2 = \sigma$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$z = \frac{68 - 67.5}{2.25\sqrt{\frac{1}{2000} + \frac{1}{1000}}}$$

$$z = \frac{0.5}{2.25(0.03873)}$$
$$z_c = 5.7377$$

Step 4:

The tabulated value for 5% level of significance of two tailed test is

$$z_t = 1.96$$

$$z_c > z_t$$

$\therefore H_0$ is rejected

Conclusion:

$\therefore$ The sample mean cannot be regarded as drawn from the same population standard deviation of 2.25

2. **Given that the information relating to two places A and B test whether there is any significant difference between two mean wages**

Solution:

Step 1:

$n_1 = 1000$ $\quad$ $n_2 = 1500$

$\bar{x}_1 = 47$ $\quad$ $\bar{x}_2 = 49$

$s_1 = 28$ $\quad$ $s_2 = 40$

Step 2:

Null hypothesis $H_0 \quad \mu_1 = \mu_2$

Alternate hypothesis $H_1 \ \mu_1 \neq \mu_2$ (two tailed test)

Step 3: Test Statistics

Here σ is not given

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

	Place A	**Place B**
Mean wages	**47**	**49**
S.D	**28**	**40**
No. of workers	**1000**	**1500**

$$z = \frac{47 - 49}{\sqrt{\frac{28^2}{1000} + \frac{40^2}{1500}}}$$

$$z = \frac{2}{1.3604}$$

$$z_c = 1.4701$$

Step 4:

The tabulated value of 5% level of significance for two tailed test z_t = 1.96

$$z_c > z_t$$

$\therefore H_0$ is rejected

Conclusion:

$\therefore$ There is no significance difference between two mean wages

3. **Two random sample of size 1600 and 2000 of farms gave a yield of 2000kg and 2050kg respectively. The variance of the farms in country may be taken as 100kg. Examine whether the two samples differ significantly in the yield.**

Solution:

Step 1:

$n_1 = 1600$ $\quad$ $n_2 = 2000$

$\bar{x}_1 = 2000$ $\quad$ $\bar{x}_2 = 2050$

$\sigma^2 = 100$ $\quad$ $\sigma = 10$

Step 2:

Null hypothesis H_0

$\mu_1 = \mu_2$

Alternate hypothesis H_1

$\mu_1 \neq \mu_2$ (two tailed test)

Step 3: Test statistics

Here σ is given

$$\sigma_1 = \sigma_2 = \sigma$$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$z = \frac{2000 - 2050}{10\sqrt{\frac{1}{1600} + \frac{1}{2000}}}$$

$$z = \frac{50}{10 \times 0.03354}$$

$$z = \frac{50}{0.3354}$$

$$z_c = 149.076$$

Step 4:

The tabulates value for two tailed test of 5% level of significance z_t = 1.96

$z_c > z_t$

$\therefore H_0$ is rejected

Conclusion:

$\therefore$ There is significance difference between samples in the yield

Type 5: Test of Significance of SD

1. If the sample S.D (or) population S.D is given then

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma^2{}_1}{2n_1} + \frac{\sigma_2{}^2}{2n_2}}}$$

2. If σ is not given

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{s^2{}_1}{2n_1} + \frac{s^2{}_2}{2n_2}}}$$

Problem:

1. **The S.D of the random sample of 900 members is 4.6 & that of another independent sample of 1600 members is 4.8 .Examine together the S.D is significantly different.**

Step 1:

<u>Given:</u>

$n_1 = 900 \;\; n_2 = 1600$

$s_1 = 4.6 \;\; s_2 = 4.8$

Step 2:

Assume null Hypothesis H_o

$\sigma_1 = \sigma_2$

Alternate Hypothesis H_1

$\sigma_1 = \sigma_2$(Two tailed test)

Step 3: Test statistics

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^{\ 2}}{2n_1} + \frac{s^2{}_2}{2n_2}}}$$

$$Z = \frac{-0.2}{\sqrt{\frac{4.6}{1800} + \frac{4.8}{3200}}}$$

Z_c=1.4526

Step 4:

The tabulated value of z at 5% level of Significance is 1.96.

Z_c<Zt

$H_o$is accepted

Conclusion:

There is no significant difference between S.D

2. A random sample drawn from the 2 countries A&B .If the following data relating to the height of the male adults

	A	**B**
Maximum height	**67.42**	**67.25**

No.of males	1000	1200
S.D	2.58	2.5

(i) If the difference of the significance of two mean

(ii) If the difference of the significance of two S.D

Solution:

Step1:

Given:

$n_1 = 1000$ $\quad$ $n_2 = 1200$

$\bar{x}_1 = 67.42$ $\quad$ $\bar{x}_2 = 67.25$

$s_1 = 2.58$ $\quad$ $s_2 = 2.5$

Step 2:

(i) Difference between S.D

Assume null Hypothesis H_o

$\sigma_1 = \sigma_2$

Alternate Hypothesis H_1

$\sigma_1 \neq \sigma_2$(two tailed test)

Step 3: Test statistics

Here σ is not given, then

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s^2{}_2}{2n_2}}}$$

$$= \frac{0.03}{\sqrt{\frac{2.58}{2000} + \frac{2.5}{2400}}}$$

=0.3894

Step 4:

The Tabulated value of z at 5% level of significance is 1.96

$Z_c < Zt$

H_0 is accepted.

Conclusion:

There is no significance difference between the two S.D.

(ii) Difference between mean

Step 1: Same

Step 2:

Assume null Hypothesis H_o

$\mu_1 = \mu_2$

Alternate Hypothesis H_1

$\mu_1 \neq \mu_2$ (two tailed test)

Step 3: Teststatistics

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}}$$

$$Z = \frac{67.42 - 67.25}{\sqrt{\frac{(2.58)^2}{1000} + \frac{(2.5)^2}{1200}}}$$

Z=1.567

Step 4:

The tablulated value of Z at 5% level of significance is 1.96

$Z_c < Zt$

$H_o$is accepted

Conclusion:

Ther is no significance difference between two mean

UNIT – V

Small Samples

STUDENT'S 't' TEST FOR SINGLE MEAN

Suppose we want to test

(a) If a random sample x_i of size n has been drawn from a normal population with a specified mean μ_o .

(b) If the sample mean differs significantly from the hypothetical value μ_o of the population mean.

In this case the statistic is given by

1. If the S.D of the samples 's' is given directly then

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}$$

2. If the SD of the sample is not given directly then

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

where $s^2 = \frac{\sum(x-\bar{x})^2}{n}$ *where* $\bar{x} = \frac{\sum x}{n}$

where $\bar{x}$ = Sample mean, μ = Population Mean,

s = standard deviation, n= sample size

Problems

1. **A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specification.**

Solution:

Step 1:

Here the sample size n=10<30.

Hence the sample is small sample.

Given $\bar{x}$ = 0.742 inches μ= 0.700 inches

S.D. s = 0.040 inches n=10

Step 2:

Null Hypothesis H_0= μ = 0.700.The product is conforming to specification

Alternative Hypothesis H_1 : μ ≠ 0.70

Step 3: The test statistic

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$$

$$t = \frac{0.742-0.700}{\frac{0.040}{\sqrt{10-1}}}$$

t = 3.15

Step 4:

Degrees of Freedom (d.f) = n – 1=10 – 1=9

The table value of t at 5% level with 9 degrees of freedom, $t_{0.05}$ = 2.26

Since calculated value of t> tabulated value of t

H_0 is rejected.

Conclusion:

Therefore, the product is not meeting the specification.

2. **The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a S.D. of 17.2. Was the advertising campaign successful?**

Solution:

Step 1:

Here the sample size n=22<30.

Hence the sample is small sample.

Here n = 22 $\bar{x}$ = 153.7

μ= 146.3 s = 17.2

Step 2:

Null Hypothesis H_0 = μ= 146.3The advertising campaign was not successful

Alternative Hypothesis H_1: μ≠146.3 (Two tailed test)

Step 3: The test statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$$t = \frac{153.7 - 146.3}{\frac{17.2}{\sqrt{22-1}}}$$

t = 1.97

Step 4:

Degrees of Freedom (d.f) = n – 1=22 – 1=21

The table value of t at 5% level of significance is 2.08

Since calculated value of t< tabulated value of t

H_0 is accepted

Conclusion:

We accept the assumption that the advertising campaign was not successful.

i.e. Advertising campaign was successful.

3. The average breaking strength of steel rods is specified to be 18.5 thousand pounds. To test this a sample of 14 rods was tested. The mean and standard deviations obtained were 17.85 and 1.955 respectively. Is the result of the experiment significant?

Solution:

Step 1:

Here the sample size n=14<30.

Hence the sample is small sample.

Given Also sample mean $\bar{x}$ = 17.85 and the population mean μ=18.5 are given.

S.D(s)= 1.955

Step 2:

Null Hypothesis H_0 : μ = 18.5 The result of the experiment is not significant

Alternative Hypothesis H_1: $\mu \neq 18.5$

Step 3: The test statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

Therefore, $$t = \frac{17.85 - 18.5}{\frac{1.955}{\sqrt{14-1}}} = -1.199$$

|t|= 1.199

Step 4:

Degrees of Freedom = n – 1 = 13

The table value of t at 5% level with 13 degrees of freedom, $t_{0.05}$ = 2.16

Since calculated value of t <tabulated value of t

we accept H_0 at 5% level.

Conclusion:

The result of the experiment is not significant.

4. A random sample of size 16 valves from a normal population showed a mean of 53 and a sum of squares of deviation from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean? Obtain 95% confidence limits of the mean of the population.

Solution:

Step 1:

Given, Sample size, n= 16

Sample Mean, $\bar{x}$ = 53

$\sum(x-\bar{x})^2$ = 150

Therefore, $s^2 = \frac{\sum(x-\bar{x})^2}{n} = \frac{150}{16} = 9.375$

S = 3.0618

Step 2:

Null Hypothesis $H_0 = \mu = 56$ The sample is taken from the population having 56 as mean.

Alternative Hypothesis $H_1 : \mu \neq 56$

Step 3: The test statistic

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$$ (Note that S.D. is not given directly)

$$t = \frac{53-56}{\frac{3.0618}{\sqrt{15}}}$$

|t| = 3.7947

Step 4:

Degrees of freedom d.f= n – 1 = 15

Since calculated t value > tabulated t value

H_0 is rejected.

Conclusion:

The sample cannot be regarded as taken from the population

The 95% confidence limit of the mean of the population is given by

$\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n}}$ = 53±2.13 x 0.79= 53±1.6827=54.68 and 51.31

Hence 95% confidence limit is [54.68,51.31].

STUDENT'S t TEST (When S.D. of the sample is not given directly)

5. A random sample of 10 boys had the following I.Q.'s of 70, 120,110,101,88,83,95,98,107,100. To these data support the assumption of a population mean of I.Q of 100? Find a reasonable range in which most of the mean I.Q values of samples of 10 boys lie.

Solution:

Step1:

Given n = 10

$$\mu = 100$$

$$\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$$

Here S.D. and mean of sample is not given directly. We have to determine these S.D. and mean as follows:

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.48
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
970		**1833.60**

Mean $\bar{x} = \sum \frac{x}{n} = \frac{972}{10} = 97.2$

We know that $s^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n} = \frac{1833.60}{10} = 183.36$

Therefore, Standard Deviation (s) = $\sqrt{183.36} = 13.541$

Step 2:

Null Hypothesis H_0= μ = 100The data support the assumption of a population mean I.Q. of 100 in the population.

Alternative Hypothesis $H_1 : \mu \neq 100$

Step 3: The test statistic

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$ (Note that S.D. is not given directly)

$$t = \frac{97.2-100}{\frac{13.541}{\sqrt{9}}} = -0.6203$$

|t| = 0.6203

Step 4:

The table value of t at 5% level of significance for 9d.f. for two tailed test is 2.26

Since calculated t value<tabulated t value

we accept the null hypothesis H_0.

Conclusion:

The data support the assumption of mean I.Q. of 100 in the population.

The 95% confidence limit of the mean of the population is given by

$\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n}}$ = 97.2±2.26 x 4.514= 97.2±10.20=107.40 and 87.00

Hence 95% confidence limit is [87.00,107.40].

6. Certain pesticide is packed into bags by a machine. A random sample of 10 bags is drawn and their contents are found to weigh (in kg) as follows:

50,49,52,44,45,48,46,45,49,45. Test if the average packing can be taken to be 50 kg. ($t_{0.05}$ for 9d.f. = 2.262)

Solution:

Step 1:

Given n = 10

$$\mu = 50$$

$$\bar{x} = \frac{\sum x}{n} = \frac{473}{10} = 47.3$$

Calculation for sample mean and S.D

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
50	2.7	7.29
49	1.7	2.89
52	4.7	22.09
44	-3.3	10.89
45	-2.3	5.29
48	0.7	0.49
46	-1.3	1.69
45	-2.3	5.29
49	1.7	2.89
45	-2.3	5.29
473		**64.1**

Mean $\bar{x} = \sum \frac{x}{n} = \frac{473}{10} = 47.3$

We know that $s^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n} = \frac{64.1}{10} = 6.41$

Therefore, Standard Deviation (s) = $\sqrt{6.14} = 2.478$

Step 2:

Null Hypothesis H_0 : μ = 50The average packing is 50 kg. i.e. μ = 50 Kg

Alternative Hypothesis H_1: $\mu \neq 50$

Step 3: The test statistic

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$$ (Note that S.D. is not given directly)

$$t = \frac{47.3-50}{\frac{2.478}{\sqrt{9}}} = -3.27$$

|t| = 3.27

Step 4:

The table value of t at 5% level of significance for 9 d.f. for two tailed test is 2.262

Since calculated t value >tabulated t value

we reject the null hypothesis H_0.

Conclusion:

The average packing is not 50 Kg.

STUDENT'S 't' TEST FOR DIFFERENCE OF MEANS

To test the significant difference between two means $\bar{x}_1$ and $\bar{x}_2$ of samples of sizes n_1 and n_2 , use the statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

where, $$s^2 = \frac{\sum(x_1-\overline{x_1})^2+\sum(x_2-\overline{x_2})^2}{n_1+n_2-2}$$

or If $S_1$and S_2 are given directly then

$$s^2 = s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$$

s_1,s_2 sample standard deviation.

Degrees of Freedom d.f. = n_1+n_2-2

1. **Samples of two types of electric light bulbs were tested for length of life and following data were obtained**

	Type I	**Type II**
Sample Number	**n_1=8**	**n_2=7**
Sample Means	**$\overline{x_1}$ =1234 hours**	**$\overline{x_2}$ = 1036 hours**
Sample S.D.	**s_1=36 hours**	**s_2= 40 hours**

Is the difference in the means sufficient to warrant that type I is superior to type II regarding the length of life.

Solution:

Step 1:

Given

n_1=8	**n_2=7**
$\overline{x_1}$ =1234 hours	**$\overline{x_2}$ = 1036 hours**
s_1=36 hours	**s_2= 40 hours**

Step 2:

Null Hypothesis H_0 : $\mu_1 = \mu_2$ The two types I and II of electric bulbs are identical.

Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$

Step 3: Test statistics

Since two sample means $\overline{x_1}$ and $\overline{x_2}$ are given and also sample standard deviation s_1 and s_2 are given directly we use the statistic

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{\left[8(36)^2 + 7(40)^2\right]}{8+7-2} = 1659.08$$

$$t = \frac{1234 - 1036}{\sqrt{1659.08\left(\frac{1}{8} + \frac{1}{7}\right)}} = 9.39$$

Step 4:

Degrees of Freedom (d.f.) = $n_1 + n_2 - 2 = 8+7 - 2 = 13$ at 5% level.

Tabulated value of t for 13 d.f. at 5% level is 2.16

Since calculated t value >tabulated t value ,

We reject the null hypothesis H0.

Conclusion:

The two types I and II of electric bulbs are not identical.

2. **To verify whether a course in accounting improved performance, a similar test was given to 12 participants both before and after the course. The marks are**

Before : 44 40 61 52 32 44 70 41 67 72 53 72

After : 53 38 69 57 46 39 73 48 73 74 60 78Was the course useful?

Solution:

Step 1:

Given

$n_1 = 12$ $n_2=12$

Step 2:

Null Hypothesis H0: There is no significant difference in the performance between before and afterthe course. i.e. μ1 = μ2

Alternatice Hypothesis H1 : $\mu 1 \neq \mu 2$

Calculation of sample means and S.D.'s

$$\bar{x} = \frac{\sum x}{12} = \frac{648}{12} = 54 \quad \bar{y} = \frac{\sum y}{12} = \frac{708}{12} = 59$$

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$	y	$y-\bar{y}$	$\left(y-\bar{y}\right)^2$
44	-10	100	53	-6	36
40	-14	196	38	-21	441
61	7	49	69	10	100
52	-2	4	57	-2	4
32	-22	484	46	-13	169
44	-10	100	39	-20	400
70	16	256	73	14	196
41	-13	169	48	-11	121
67	13	169	73	14	196
72	18	324	74	15	225
53	-1	1	60	1	1

72	18	324	78	19	361
		2176			**2250**

$$\sum\left(x-\bar{x}\right)^2 = 2176 \qquad \sum\left(y-\bar{y}\right)^2 = 2250$$

Therefore,

$$s^2 = \frac{\left[\sum\left(x-\bar{x}\right)^2 + \sum\left(y-y\right)^2\right]}{n_1+n_2-2}$$

$$= \frac{\left[2176+2250\right]}{12+12-2} = \frac{4426}{22} = 201.18$$

Therefore, s=14.18

Now,

$$t = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{54-59}{14.18\sqrt{\frac{1}{12}+\frac{1}{12}}} = -\ 0.8637$$

i.e. Calculated |t| = 0.8637

Step 4:

Degrees of freedom = n_1+n_2-2 = 12+12 – 2 = 22

Therefore, tabulated t for 22 d.f. at 5% level is 2.07.

Since calculated t< tabulated t

we have to accept the null hypothesis.

Conclusion:

The course is useful.

3. Below are given the gain in weights (in lbs) of pigs fed of two diets A and B.

Diet A	**25**	**32**	**30**	**34**	**24**	**14**	**32**	**24**	**30**	**31**	**35**	**25**	**-**	**-**	**-**
Diet B	**44**	**34**	**22**	**10**	**47**	**31**	**40**	**30**	**32**	**35**	**18**	**21**	**35**	**29**	**22**

Test if the two diets differ significantly as regards their effect on increase in weight.

Solution:

Step 1:

$n_1 = 12$ $n_2=15$

Step 2:

Null Hypothesis H0 = There is no significant difference between the mean increase in weight dure to diets A and B. i.e. μ1 = μ2.

Alternative Hypothesis H1: μ1 ≠ μ2

Step 3:

$$\bar{x} = \frac{\sum x}{n_1} = \frac{336}{12} = 28 \quad \bar{y} = \frac{\sum y}{n_2} = \frac{450}{15} = 30$$

Calculation for sample means and S.D.s

x_1	$x_1 - \bar{x}_1$	$\left(x_1 - \bar{x}_1\right)^2$	x_2	$x_2 - \bar{x}_2$	$\left(x_2 - \bar{x}_2\right)^2$
25	-3	9	44	14	196
32	4	16	34	4	16
30	2	4	22	-8	64
34	6	36	10	-20	400
21	-4	16	47	17	289
14	-14	196	31	1	1
32	4	16	40	10	100
24	-4	16	30	0	0
30	2	4	32	2	4
31	3	9	35	5	25
35	7	49	18	-12	144

25	-3	9	21	-9	81
			35	5	25
			29	-1	1
			22	-8	64
336		**380**	**450**		**1410**

$$\sum\left(x_1-\bar{x}_1\right)^2=380 \qquad \sum\left(x_2-\bar{x}_2\right)^2=1410$$

Therefore,

$$s^2=\frac{\left[\sum\left(x_1-\bar{x}_1\right)^2+\sum\left(x_2-\bar{x}_2\right)^2\right]}{n_1+n_2-2}=\frac{[380+1410]}{12+15-2}=71.6$$

$$t=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}=\frac{28-30}{\sqrt{71.6\left(\frac{1}{12}+\frac{1}{15}\right)}}=-0.609$$

i.e. Calculated value of |t| = 0.609

Step 4:

Degrees of freedom = n_1+n_2-2 = 12+15 – 2 = 25

Tabulated t for 25 d.f. at 5% level = 2.06.

Since calculated t< tabulated t

we accept the null hypothesis H_0.

Conclusion:

There is no significant difference between the mean increase in weight die to diets A &B

4. **The mean of two random sample sizes 9 and 7 are 196.42 and 198.82. The sum of the squares of the deviations from the mean are 26.94 and 18.73 respectively. can the sample be considered drawn from normal populations of equal means.**

Solution:

Step 1:

Given

$n_1 = 9$	$n_2 = 7$
$\overline{x_1} = 196.42$	$\overline{x_2} = 198.82$
$\sum(x_1 - \overline{x}_1)^2 = 26.94$	$\sum(x_2 - \overline{x}_2)^2 = 18.72$

Step 2:

Null Hypothesis H_0: $\mu_1 = \mu_2$

Alternative Hypothesis H_1: $\mu_1 \neq \mu_2$

Step 3:

$$t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{196.42 - 198.82}{1.8059\sqrt{\left(\frac{1}{9} + \frac{1}{7}\right)}}$$

t = 2.6371

Step 4:

Degree of freedom = $n_1 + n_2 - 2 = 9 + 7 - 2 = 14$

At 5% L.O.S for t test the table value is 2.15

Therefore t calculated value > t tabulated value

Therefore H_0 is rejected

Conclusion:

The sample can not be considered as drawn from the same normal population

F – TEST FOR VARIANCE

To test whether if there is any significant difference between two estimates of population variance (or) To test if the two samples have come from the same population, we use F-Test.

In this case we set up null hypothesis $H_0 = \sigma 1 = \sigma 2$. i.e. Population variances are same.

Under H_0, the test statistic is

1. If S_1^2 is greater, then

$$F = \frac{S_1^2}{S_2^2}$$

2. If S_2^2 is greater, then

$$F = \frac{S_2^2}{S_3^2}$$

Where $S_1^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n_1 - 1}$, $S_2^2 = \frac{\sum\left(y-\bar{y}\right)^2}{n_2 - 1}$

The degrees of freedom are $\gamma 1 = n_1 - 1$, $\gamma 2 = n_2 - 1 = (n_1 - 1, n_2 - 1)$

Note:We will take greater of the variances S_1^2 or S_2^2 in the numerator and adjust for the degrees of freedom accordingly.i.e. $F = \frac{GreaterVariance}{SmallerVariance}$

If sample variance or sample SD are given directly then

$$F = \frac{n_1 s_1^2 / n_1 - 1}{n_2 s_2^2 / n_2 - 1}$$

1. **In one sample of 10 observations from a normal population, the sum of the squares of the deviations of the sample values from the sample mean is 102.4 and in another sample of 12 observations from another normal population, the sum of the squares of the deviations of the sample values from the sample mean is 120.5. Examine whether the two normal populations have the same variances.**

Solution:

Step 1:

Given $n_1=10$ $n_2=12$

$\sum\left(x-\bar{x}\right)^2 = 102.4$ $\sum\left(y-\bar{y}\right)^2 = 120.5$

Therefore,

$$S_1^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n_1 - 1} \qquad S_2^2 = \frac{\sum\left(y-\bar{y}\right)^2}{n_2 - 1}$$

$$= \frac{102.4}{9} \qquad = \frac{120.5}{11}$$

$$= 11.37 \qquad = 10.95$$

Step 2:

Null Hypothesis H_0: The two normal populations have the same variances.

$$\sigma_1^2 = \sigma_2^2$$

Alternative Hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$

Step 3: The test statistic

$$F = \frac{S_1^2}{S_2^2}\left[\because S_1^2 > S_2^2\right] = \frac{11.37}{10.95} = 1.038$$

i.e. Calculated F = 1.132

Step 4:

Tabulated value of F for (9,11) d.f. at 5% level of significance is 2.91 (approx).

Since calculated value of F < tabulated F

we accept the null hypothesis H_0.

Conclusion:

The two normal populations have the same variances.

2. Two random samples gave the following results:

Sample	Size	Sample Mean	Sum of Squares of Deviation from the Mean
1	**10**	**15**	**90**
2	**12**	**14**	**108**

Test whether the samples come from the same normal population.

Solution:

Step 1:

Given

n_1= 10	$\overline{x_1} = 15$	$\sum\left(x_1 - \overline{x_1}\right)^2 = 90$
n2 = 12	$\overline{x_2} = 14$	$\sum\left(x_2 - \overline{x_2}\right)^2 = 108$

Step 2:

Null Hypothesis H_0: The two samples have been drawn from the same normal population.

i.e. $H_0 = \mu 1 = \mu 2$ and $\sigma_1^2 = \sigma_2^2$

Here we have to use two tests (i) To test equality of variances by F-test (ii) To test equality of means by t-test.

(i) F – test (Equality of Variances)

Step 3:

Given, n1=10 n2=12

$\overline{x_1} = 15$ $\overline{x_2} = 14$

$\sum\left(x_1 - \overline{x_1}\right)^2 = 90$ $\sum\left(x_2 - \overline{x_2}\right)^2 = 108$

$$S_1^2 = \frac{\sum\left(x_1 - \overline{x_1}\right)^2}{n_1 - 1} = \frac{90}{9} = 10 \qquad S_2^2 = \frac{\sum\left(x_2 - \overline{x_2}\right)^2}{n_2 - 1} = \frac{108}{11} = 9.82$$

we use $F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.018$

i.e. Calculated F=1.018

Step 4:

Tabulated F at 5% level for (9,11) d.f. is 2.90. i.e. $F_{0.05}(9,11) = 2.90$

Since Calculated F < Tabulated F

we accept the null hypothesis H_0. i.e. μ1=μ2

Conclusion:

The samples come from the same normal population.

(ii) t-test (To Test Equality of Means)

Step 2: Null Hypothesis H0: μ1=μ2

Given, n1=10 n2=12

$\overline{x_1} = 15$ $\overline{x_2} = 14$

$\sum\left(x_1 - \overline{x_1}\right)^2 = 90$ $\sum\left(x_2 - \overline{x_2}\right)^2 = 108$

$$s^2 = \frac{\left[\sum\left(x_1 - \overline{x_1}\right)^2 + \sum\left(x_2 - x_2\right)^2\right]}{n_1 + n_2 - 2} = \frac{[90 + 108]}{10 + 12 - 2} = 9.9$$

$\therefore s = 3.15$

$$t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{15 - 14}{3.15\sqrt{\frac{1}{10} + \frac{1}{12}}} = 0.74$$

,

Therefore, Calculated value of t = 0.74

Step 4:

Tabulated value of t for 20 d.f. (n1+n2-2) at 5% level of significance is 2.086.

Since calculated value of t < tabulated value of t.

We accept the null hypothesis H0. i.e. μ1=μ2.

Conclusion:

Hence from (i) and (ii) , the given samples have been drawn from the same normal population.

3. It is known that the mean diameters of rivets produced by two firms A and B are practically the same but the standard deviations may differ. For 22 rivets produced by the firm A, the standard deviation is 2.9 mm, while for 16 rivets manufactured by firm B, the standard deviation is 3.8 mm. Compute the statistic you would use to test whether the products of firm A have the same variability as those of firm B and test it's significance

Solution:

Given $n_1 = 22 \;\; n_2 = 16$

$s_1 = 2.9 \;\; s_2 = 3.8$

Here the S.D.'s of the samples s1 and s2 are given. Therefore, the population variance S_1^2 and S_2^2 are obtained by using the relations,

Step 2:

Null Hypothesis H0: The products of both the firms A and B have the same variability.

i.e. $\sigma_1^2 = \sigma_2^2$

Alternative hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$

Step3: the test statistic

$$F = \frac{n_2 s_2^2 / n_2 - 1}{n_1 s_1^2 / n_1 - 1}$$

$$F = \frac{16 \times (3.8)^2 / 15}{22 \times (2.9)^2 / 21}$$

$$F = 1.748$$

i.e. Calculated F = 1.75

Step 4:

Tabulated value of F with (15,21) d.f. at 5% of significance is 2.20.

Since calculated F < tabulated F

we accept the null hypothesis H_0.

Conclusion:

The products of both firms A and B have the same variability.

Chi square test for goodness of fit and independence of attributes

1. The following table gives number of aircrafts accidents that occurred during various days. Use Ψ^2 test and find whether the accidents are uniformly distributed over the week

Days :	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
No of Accidents:	14	16	8	12	11	9	14

Solution:

Step 1:

Given

n = 7

Step 2:

Null hypothesis H_0 : There are no significance difference between O_i and E_i

Step 3:

$$E = \frac{\sum o}{n} = \frac{84}{7} = 12$$

$$\chi^2 = \sum \left[\frac{(O-E)^2}{E} \right]$$

O	E	$O-E$	$\frac{(O_i - E_i)^2}{E_i}$
14	12	4	0.333
16	12	16	1.333
8	12	16	1.333
12	12	0	0
11	12	1	0.0835
9	12	9	0.75
14	12	4	0.333

$$\chi^2 = \sum\left[\frac{(O-E)^2}{E}\right] = 4.165 \text{and}$$

Step 4:

DOF= n – 1 = 7 – 1=6

At 5% $\Psi^2{}_{tab} = 12.592$

$\Psi^2{}_{ca;} < \Psi^2{}_{tab}$. H_0 is accepted.

Conclusion:

There are no significance difference between O_i and E_i

2)The theory predicts the proportion of beams in the 4 groups A,B,C,D be 9:3:3:1. In an experiment with 1600 beams the numbers in 4 groups where 882,313,287,118. Does the experimental result support the theory.

Solution:

Step 1:

Given n = 4

Step 2:

Null hypothesis H_0 : There is no significance difference between O and E

Step 3:

If we divide 1600 in the ratio 9:3:3:1 We get expected frequency

$$E = \frac{total\ no\ of\ product}{sum\ of\ ratio} \times Individual\ ratio$$

E_i (A)=$\frac{1600}{16} X9 = 900$

E_i (B)=$\frac{1600}{16} X3 = 300$

E_i (C)=$\frac{1600}{16} X3 = 300$

E_i (D)=$\frac{1600}{16} X1 = 100$

O	E		(O-E)	$\frac{O-E}{E}$
882	900		324	0.360
313	300		169	0.563
287	300		169	0.563
118	100		324	3.24
				4.726

$$\Psi^2{}_{cal} = \sum\left[\frac{(O-E)^2}{E}\right] = 4.726$$

Step 4:

D.O.F = 4 – 1= 3

$\Psi^2{}_{tab}$ at 5% los=7.815

H_0 is accepted.

Conclusion:

Thus, the experimental result supports the theory.

3) **The survey of 320 families with 5 childrens each gave the following distribution:**

No of boys	**5**	**4**	**3**	**2**	**1**	**0**
No of girls	**0**	**1**	**2**	**3**	**4**	**5**
No of families	**14**	**56**	**110**	**88**	**40**	**12**

Use Ψ^2 test whether the male and female birth are equally popular.

<u>Solution:</u>

Step 1:

Given N=320n=5

Observed frequency	14	56	110	88	40	12

Step 2:

Null hypothesis H_0 :The female and male birth are equally popular

Step 3:

We use binomial distribution to find the expected frequency.

Expected frequency are given by f(x)=N .n $C_n\, p^x . q^{n-x}$

Where N=320; n=5

p=probability of getting a girl $=\frac{1}{2}$

q=1-$\frac{1}{p}$ = 1-$\frac{1}{2}$ =$\frac{1}{2}$

x=0,1,2,3,4,5

f(x) = N . nCx$p^x q^{n-x}$

f(0)= 320 . $5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$ =10

f(1)= 320 . $5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4$=50

f(2)= 320 . $5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3$ =100

f(3)= 320 . $5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$ =100

f(4)= 320 . $5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1$ =50

f(5)= 320 . $5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$ = 10

O	14	56	110	88	40	12
E	10	50	100	100	50	10

O	E	O-E	$\frac{(O-E)^2}{E}$
14	10	16	1.6
56	50	36	0.72
110	100	100	1

88	100	144	1.44
40	50	100	2
12	10	4	0.4
			7.16

$\Psi^2{}_{cal} = \frac{(O-E)^2}{E} = 7.16$

Step 4:

D.O.F = 6 – 1=5

$\Psi^2{}_{tab}$ at5% cal = 11.07

$\Psi^2{}_{cal} > \Psi^2{}_{tab}$

H_0 isaccepted.

Conclusion: **t**herefore, the male and female are equally popular.

4)Examine the goodness of fit for the Poisson distribution to the following data.

x	**0**	**1**	**2**	**3**	**4**
f	**109**	**65**	**22**	**3**	**1**

Solution:

Step 1:

Given n = 4

Step 2:

Null hypothesis H_0 : there is no significance difference between O and E

Step 3:

To find expected frequency :

f(x) = N $e^{-\lambda}\frac{\lambda^x}{x!}$

Here , mean = variance = λ

To find λ:

x	f	fx
0	109	0
1	65	65

2	22	44
3	3	9
4	1	4
	Σf = 200	Σfx = 122

Mean $=\frac{\Sigma fx}{\Sigma f}$

$\lambda = 0.61$

Here, N $=\Sigma$f = 200

f(x) = N . $e^{-\lambda}\frac{\lambda^x}{x!}$

f(0) = 200 . $e^{-0.61}\frac{(0.61)^0}{0!}$= 108.6701

f(1) = 200 . $e^{-0.61}\frac{(0.61)^1}{1!}$= 66.2888

f(2) = 200. $e^{-0.61}\frac{(0.61)^2}{2!}$= 20.2181

f(3) = 200 . $e^{-0.61}\frac{(0.61)^3}{3!}$= 4.111

f(4) = 200 . $e^{-0.61}\frac{(0.61)^4}{4!}$= 0.6269

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
109	108.6701	0.1088	1.0015 x 10^{-3}
65	66.2888	1.661	0.0251
22	20.2181	3.1751	0.157
3	4.111	1.2343	0.3002
1	0.6269	0.1392	0.2221
			0.7054

$\Sigma[\frac{(O-E)^2}{E}]$=0.7054

Therefore,$\Psi^{2\ cal}$ =0.7054

Step 4:

D.O.F = n – 2= 5 – 2= 3

$\Psi^2{}_{tab}$ at5% cal = 7.815

$\Psi^2{}_{cal} < \Psi^2{}_{tab}$

Hence H_0 is accepted.

Conclusion:

There is no significance difference between O and E

5)From the following information state whether the condition of the home is associated with the condition of the child .

Condition of Home

Condition Of Child

	Clean	**Dirty**
Clean	**69**	**51**
Fairly clean	**81**	**20**
Dirty	**35**	**44**

Solution:

Step 1:

Null hypothesis H_0 : The condition of the home is associated with the condition of the child .

Step 2:

Given O : 69 51 81 20 35 44

Condition of Home

Condition Of Child

	Clean	Dirty	Total
Clean	69	51	120
Fairly clean	81	20	101
Dirty	35	44	79

$E(69) = \frac{120 \ X \ 185}{300} = 74$

$E(51) = \frac{120 \ X \ 115}{300} = 46$

E(81) = $\frac{101\ X\ 185}{300}$ = 62.283

E(20) = $\frac{101\ X\ 115}{300}$ = 38.717

E(35) = $\frac{79\ X\ 185}{300}$ = 48.717

E(44) = $\frac{79\ X\ 115}{300}$ = 30.283

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
69	74	25	0.3378
51	46	25	0.3435
81	62.283	350.3261	5.6247
20	38.717	350.3261	9.0484
35	48.717	188.1561	3.8622
44	30.283	188.1561	6.2133

$\Psi^2{}_{cal}$ = 25.6299

Step 3:

D.O.F = (m-1) (n-1) = (3-1) (2-1) = 2

$\Psi^2{}_{tab}$ at 5% cal = 5.991

$\Psi^2{}_{cal} > \Psi^2{}_{tab}$

H_0 isrejected.

Conclusion:

The condition of the home is associated with the condition of the child .

6)In an experiment of immunization of cattle from TB , The following results were

	Affected	**Non-affected**
Inocalated	**12**	**28**
Non – Inocalated	**13**	**7**

Examine the effect of vaccine controlling the incidents of TB

Solution:

Step 1:

Null hypothesis H_0 : The effect of vaccine controlling the incidents of TB

Step 2:

O	12	28	13	7

To find Expected frequency:

	Affected	Non-affected	
Inoculated	12	28	40
Non – Inoculated	13	7	20
	25	35	60

E(12) = $\frac{40\ X\ 25}{60}$ = 16.6667

E(13) = $\frac{25\ X\ 20}{60}$ = 8.3333

E(28) = $\frac{35\ X 40}{60}$ = 23.3333

E(7) = $\frac{35\ X\ 20}{60}$ = 11.6667

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
12	16.6667	21.7781	1.3067
28	23.3333	21.7781	0.9333
13	8.3333	21.7781	2.6134
7	11.6667	21.7781	1.8667

$\Psi^2{}_{cal} = 6.7201$

D.O.F = (m – 1) (n – 1) = 1

$\Psi^2{}_{tab}$ at5% cal = 3.841

$\Psi^2{}_{cal} > \Psi^2{}_{tab}$

H_0 isrejected.

Conclusion:

The effect of vaccine controlling the incidents of TB

7)Fit a binomial distribution for the following data and also test the goodness of fit.

X	**0**	**1**	**2**	**3**	**4**
F	**5**	**29**	**36**	**25**	**5**

Solution:

Step 1:

Given:

O	5	29	36	25	5

Step 2:

Null hypothesis H_0 : There is no significance difference between O and E

Step 3: To find expected frequency

f(x) = N . $nc^x\ p^x q^{n-x}$

Here, n = 4

To find p : In binomial distribution ;

X	f	fx
0	5	0
1	29	29
2	36	72
3	25	75
4	5	20
	100	196

Mean = $\frac{\sum fx}{\sum f}$ = n p

Mean = $\frac{196}{100} = np$

n p = 1.96

4 p = 1.96

p = 0.49

q = 1 – p = 0.15

f(x) = N . $nc^x\ p^x q^{n-x}$

f(0) = 100 . $4c^0\ (0.49)^0(0.51)^4$= 6.7652

f(1) = 100 . $4c^1\ (0.49)^1(0.51)^3$ = 25.9995

f(2) = 100. $4c^2\ (0.49)^2(0.51)^2$ = 37.470

f(3) = 100. $4c^3\,(0.49)^2(0.51)^1 = 24.0004$

f(4) = 100 . $4c^4\,(0.49)^2(0.51)^1$ = 5.7648

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
5	6.7052	3.1159	0.4606
29	25.9995	9.0030	0.3463
36	37.47	2.1609	0.0577
25	24	1	0.0417
5	5.7648	0.5849	0.1014
			1.0077

$$\Psi^2{}_{cal} = 1.0077$$

Step 4:

D.O.F = n-1 = 5-1 = 4

$\Psi^2{}_{tab}$ at 5% ; los = 9.488

$\Psi^2{}_{tab} > \Psi^2{}_{cal}$

Hence, H_0 is accepted.

Conclusion: Therefore , there is no significant difference between O and E.

8)The following data are for a sample of 300 car owners. Who where classified with respect to age and the number of accidents they had during the past 2 years . Test whether there is any relationship between these 2 variables.

Number of Accidents

		1	**1 or 2**	**More than 2**
Age	**Less than 30**	**8**	**23**	**14**
	Between 30 and 50	**21**	**42**	**12**
	More than 50	**71**	**90**	**19**

Solution:

Step 1:

Null hypothesis H_0 There is relationship between these 2 variables

O	8	23	14	21	42	12	71	90	19

Step 2:To find expected frequency :

	1	1 or 2	More than 2	
Less than 30	8	23	14	45
Between 30 and 50	21	42	12	75
More than 50	71	90	19	180
	100	155	45	N = 300

E(8) = $\frac{45\ X\ 100}{300}$ = 15

E(23) = $\frac{45\ X\ 155}{300}$ = 23.25

E(14) = $\frac{45\ X\ 45}{300}$ = 6.75

E(21) = $\frac{75\ X\ 100}{300}$ = 25

E(42) = $\frac{75\ X\ 155}{300}$ = 38.75

E(12) = $\frac{75\ X\ 45}{300}$ = 11.25

E(71) = $\frac{180\ X\ 100}{300}$ = 60

E(90) = $\frac{180\ X\ 155}{300}$ = 93

E(19) = $\frac{180\ X\ 45}{300}$ = 27

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
8	15	49	3.2667
23	23.25	0.0625	0.0027
14	6.75	52.5625	7.787
21	25	16	0.64
42	38.75	10.5625	0.2756
12	11.25	0.5625	0.05
71	60	121	2.0167
90	93	9	0.0968
19	27	64	2.3704

$\Psi^2{}_{cal}$ = 16.5059

Step 4:

D.O.F = (m-1) (n-1)

= (3-1) (3-1) = 4

$\Psi^2{}_{tab}$ at5 % ; los = 9.488

Hence, $\Psi^2{}_{cal} > \Psi^2{}_{tab}$

Therefore, H_0 is rejected.

Conclusion:

There is relationship between these 2 variables

PART- A QUESTION AND ANSWERS

UNIT I

PARTIAL DIFFERENTIAL EQUATIONS

1. **What is a partial differential equation (PDE)?**

   A PDE is an equation involving a function of two or more variables and some of its partial derivatives.

   (e.g.) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y}$, $\left(\frac{\partial u}{ox}\right)^3 + \frac{\partial u}{\partial y} = 0$

2. **Write the standard notations used to denote partial derivatives.**

If z is taken as a dependent variable and x, y are taken as independent variables such that z = f(x, y), then we use the following notations for partial derivatives.

$$\frac{\partial z}{\partial x} = p; \quad \frac{\partial z}{\partial y} = q, \quad \frac{\partial^2 z}{\partial x^2} = r, \quad \frac{\partial^2 z}{\partial x \partial y} = s, \quad \frac{\partial^2 z}{\partial y^2} = t$$

3. **Define order of PDE.**

   The order of a PDE is the order of the highest derivative occurring in it.

4. **Form partial differential equation by eliminating the arbitrary constants from**

$z = (x^2 + a)(y^2 + b)$. (Nov 2014)

Given that $z = (x^2 + a)(y^2 + b)$ … (1)

$$\frac{\partial z}{\partial x} = p = 2x(y^2 + b) \Rightarrow y^2 + b = p/2x \quad \ldots(2)$$

$$\frac{\partial z}{\partial y} = q = 2y(x^2 + a) \Rightarrow x^2 + a = q/2y \quad \ldots(3)$$

Substitute (2) & (3) in (1) we get

$$z = \frac{p}{2x}.\frac{q}{2y}$$

$\boxed{4xyz = pq}$ which gives the required partial differential equation.

5. Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2, z - xy) = 0$ **APR (2013)**

Given that $f(u, v) = 0$ ... (1)

Where $u = x^2 + y^2$, $v = z - xy$

The elimination of 'f' from (1) gives $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$

(i.e.) $\begin{vmatrix} 2x & p-y \\ 2y & q-x \end{vmatrix} = 0$

(i.e.) $2x(q - x) - (p - y)2y = 0$

$\boxed{py - qx = y^2 - x^2}$ which gives the required partial differential equation.

6. Find PDE by eliminating the arbitrary function 'f' from $f(x^2 + y^2) = y^2 + z^2$.

Given that $f(x^2 + y^2) = y^2 + z^2$ … (1)

$$\frac{\partial f}{\partial x} 2x = 2z \frac{\partial z}{\partial x}$$

$$xf' = zp \Rightarrow f' = \frac{pz}{x} \quad \ldots(2)$$

$$\frac{\partial f}{\partial y}2y = 2y + 2z\frac{\partial z}{\partial y}$$

$$yf' = y + zq$$

$$f' = \frac{y+zq}{y} = 1 + \frac{zq}{y} \qquad ...(3)$$

From (2) & (3) $1 + \frac{zq}{y} = \frac{pz}{x}$

$\boxed{(y + zq)\ x = pzy}$ which gives the required partial differential equation.

7. Eliminate'*f* 'from z = x + y + f(xy) to form partial differential equation.

Given that z = x + y + f(xy) …. (1)

$$\frac{\partial z}{\partial x} = 1 + f'(xy)y$$

$$p = 1 + yf' \Rightarrow f' = \frac{p-1}{x} \qquad \ldots(2)$$

$$\frac{\partial z}{\partial y} = 1 + f'(xy)x$$

$$q = 1 + xf' \Rightarrow f' = \frac{q-1}{x} \qquad \ldots(3)$$

From (2) & (3) $\boxed{\frac{p-1}{y} = \frac{q-1}{x}}$ which gives the required PDE.

8. Find the PDE of all planes through the origin.

The general equation of a plane is ax + by + cz + d = 0 …(1)

Since it passes through origin, we have

a(0) + b(0) + c(0) + d = 0 $\Rightarrow$ d = 0

Therefore (1) becomes ax + by + cz = 0 ….(2)

$$a + c\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{-a}{c} \Rightarrow p = \frac{-a}{c} \text{ ...(3)}$$

$$b + c\frac{\partial z}{\partial y} = 0$$

$$b + cq = 0 \Rightarrow \quad q = \frac{-b}{c} \text{ …(4)}$$

From (3) & (4) we get a = -cp

b = -cq

Substitute a, b in (2), -cpx – cqy + cz = 0

$\boxed{z = px + qy}$ which gives the required partial differential equation.

9.Find PDE of all spheres of radius c having their centres in the xoy plane.

Let the centre of the sphere be (a, b, 0), a point on the xoy plane.

Given that radius = c.

∴ Therefore the equation of sphere is$(x - a)^2 + (y - b)^2 + z^2 = c^2$ …(1)

Here a, b are the arbitrary constants, c is given.

$2(x - a) + 2zp = 0$ ... (2)

$2(y - b) + 2zq = 0$ ... (3)

From (2), x – a = - pz

(3), y – b = - qz

Substituting in (1), we get

$p^2z^2 + q^2z^2 + z^2 = c^2$

$\boxed{z^2\left(p2+q^2+1\right) = c^2}$ which is the required PDE.

10. Form the PDE by eliminating the constants from $(x-a)^2 + (y-b)^2 + z^2 = 1$.**(Nov-2012)**

Given that $(x-a)^2 + (y-b)^2 + z^2 = 1$

…(1)

Here a, b are the arbitrary constants.

$2(x-a) + 2zp = 0$ ... (2)

$2(y-b) + 2zq = 0$ ... (3)

From (2), $x - a = -pz$

(3), $y - b = -qz$

Substituting in (1), we get

$p^2z^2 + q^2z^2 + z^2 = 1$

$\boxed{z^2\left(p^2+q^2+1\right) = 1}$ which is the required PDE.

11. Find the PDE of all spheres whose centers lie on the z – axis.

Let the centre of the sphere be (0, 0, c), a point on the z – axis and r its radius.

$\therefore$ The equation of sphere is $(x-0)^2 + (y-0)^2 + (z-c)^2 = r^2$

$x^2 + y^2 + (z-c)^2 = r^2$ …(1)

$$2x + 2(z-c)\frac{\partial z}{\partial x} = 0 \Rightarrow x + (z-c)p = 0$$

$$(z-c) = \frac{-x}{p} \quad ...(2)$$

$$2y + 2(z-c)\frac{\partial z}{\partial y} = 0 \Rightarrow y + (z-c)q = 0$$

$$z - c = \frac{-y}{q} \qquad \ldots.(3)$$

From (2) & (3), we get $\frac{-x}{p} = \frac{-y}{q}$

$\boxed{py = qx}$ which gives the required PDE.

12. What is the difference between ordinary and partial differential equation?

Ordinary differential equation	**Partial differential equation**
The orders will be same, as the number of constants eliminated	The order will be one when the number of constants is equal to number of independent variable

13. Write the three types of solution of a PDE.

(i) A solution which contains maximum possible number of arbitrary constants is called a complete integral

(ii) A solution obtained by giving particular values to the arbitrary constants in a complete integral is called a particular integral.

(iii) A solution which contains maximum possible number of arbitrary functions is called a general integral.

14. Define singular solution.

Let f(x, y, z, p, q) = 0 be a PDE.

Let the complete solution of this equation be $\phi(x, y, z, a, b) = 0$ ... (1)

Differentiate partially w.r.t. 'a' and 'b'.

$$\frac{\partial \phi}{\partial a} = 0, \qquad \ldots(2)$$

$$\frac{\partial \phi}{\partial b} = 0 \qquad \ldots.(3)$$

Eliminate ‘a’ and ‘b’ from (2) & (3).

The eliminant of ‘a’ and ‘b’ is called singular integral.

15. Solve : $\frac{\partial z}{\partial x} = \sin x$

Given that $\frac{\partial z}{\partial x} = \sin x$

Integrating w.r.t. ‘x’, we get

z = - cosx + f(y)

∴ The solution is z = -cosx + f(y) where f is arbitrary.

16. Find the general solution of $\frac{\partial^2 z}{\partial y} = 0$

Given that $\frac{\partial^2 z}{\partial y^2} = 0$

Integrating w.r.t. ‘y’

$\frac{\partial z}{\partial y} = f(x)$ where f is arbitrary.

Integrating w.r.t. ‘y’

z = y f(x) + g(x) where g is arbitrary.

∴ $\boxed{z = y\,f(x) + g(x)}$ is the general solution.

17. Find the complete integral of pq = 4

Let z = ax + by + c be the solution … (1)

$$\frac{\partial z}{\partial x} = p = a$$

$$\frac{\partial z}{\partial y} = q = b$$

Substitute p = a, q = b in pq = 4

$$ab = 4$$

$$a = \frac{4}{b}$$

(1) becomes $\boxed{z = \frac{4}{b}x + by + c}$

This gives the complete integral, since it contains only two arbitrary constants.

18. Define clairaut's type of equation and explain how to solve it.

The PDE of the type z = px + qy + f(p, q) ...(1)

is called as clairant's equation.

Let z = ax + by + f(a,b) be the solution of (1)

This is the complete integral of (1)

$$\Rightarrow p = a,\ q = b$$

Substituing in (1), we get

z = ax + by + f(a, b)

$$\frac{\partial z}{\partial a} = x + f'(a,b) = 0 \Rightarrow f'(a,b) = -x$$

$$\frac{\partial z}{\partial b} = y + f'(a,b) = 0 \Rightarrow f'(a,b) = -y$$

$\therefore$ -x = -y

(i.e.) $\boxed{x = y}$ which is the singular integral of (1)

19. Write the complete integral of p + q = x + y

Given that p + q = x + y …. (1)

Let p – x = - (q – y) = a, an arbitrary constant.

Then p = x + a, q = y – a

We know that $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$$dz = pdx + qdy$$

$$dz = (x+a)dx + (y-a)dy$$

Integrating on both sides, we get $\boxed{z=\frac{x^2}{2}+ax+\frac{y^2}{2}-ay+b}$ where a, b are arbitrary constants. This gives the complete integral of (1)

20. Write the general solution of pyz +qzx = xy

The given PDE is of the form Pp + Qq = R

Where P = zy ; Q = zx ; R = xy

Lagrange's subsidiary equations are

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$$

(i.e.) $\frac{dx}{yz}=\frac{dy}{zx}=\frac{dz}{xy}$ …. (1)

Taking first two ratios in (1), we get

$$\frac{dx}{y\cancel{z}}=\frac{dy}{\cancel{z}x}\Rightarrow\int xdx=\int ydy$$

(i.e.) $x^2-y^2=c_1$

Taking second and third ratio, we get, $\frac{dy}{z\cancel{x}}=\frac{dz}{\cancel{x}y}\Rightarrow\int ydy=\int zdz$

(i.e.) $y^2-z^2=c_2$

Hence the general solution is $\boxed{\phi\left(x^2-y^2,y^2-z^2\right)=0}$ where ϕ is arbitrary.

21. Solve : $\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+4\frac{\partial^2 z}{\partial y^2}=0$

Let $\frac{\partial}{\partial x}=D$ and $\frac{\partial}{\partial y}=D'$

$\therefore$ The given equation becomes $(D^2-4DD'+4D'^2)z=0$

Put D = m, $D^1 = 1$.

The eauxillary equation is $m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

m = 2, 2

$\therefore$ The solution is $\boxed{z=f_1(y+2x)+x\,f_2(y+2x)}$ where f_1, f_2 are arbitrary functions.

22. Solve : $(D^2 + 3DD' + 2D'^2)\,z = 0$

Put D = m, $D^1 = 1$

The auxillary equation is $m^2 + 3m + 2 = 0$

$$(m + 2)(m + 1) = 0$$

$$m = -2, -1$$

$\therefore$ The solution is $\boxed{z = f_1(y - 2x) + f_2(y - x)}$ where f_1 and f_2 are arbitrary functions.

23. Find the particular integral of $(D^2 - 4DD' + 3D'^2)\, z = e^{x+y}$

$$P.I = \frac{1}{D^2 - 4DD^1 + 3D^{12}} e^{x+y}$$

Put D = 1, $D^1 = 1$

$$= \frac{1}{0} e^{x+y}$$

$\therefore$ Multiply by x and differentiate the denominator w.r.t. D.

$$P.I. = x \frac{1}{2D - 4D^1} e^{x+y}$$

$$= x \frac{1}{2-4} e^{x+y} \quad \{ \text{ by putting } D = 1,\ D^1 = 1 \}$$

$$\therefore \boxed{\text{P.I.} = \frac{-xe^{x+y}}{2}}$$

24. Solve : $(D^3 - 3DD'^2 + 2D'^3)\, z = 0$ **(Nov-2012)**

Put D = m, $D^1 = 1$

The auxillary equation is $m^3 - 3m + 2 = 0$

	1	0	-3	2
1				
	0	1	1	-2

m=1 is a root

1	1	-2	0

The other roots are given by $m^2 + m - 2 = 0$

$$(m + 2)(m - 1) = 0$$

$$m = -2, 1$$

Therefore the roots are m = 1,1,-2

$\therefore$ The solution is $\boxed{z = f_1(y - 2x) + f_2(y + x)} + xf_3(y + x)$ where f_1 and f_2 are arbitrary functions.

25. Define Lagrange's linear equations.

The equation of the form Pp + Qq = R is known as Lagrange's equation, where P, Q and R are the functions of x, y and z. To solve this PDE, it is enough to solve $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ which is called subsidiary equations. If its solution is of the form u(x, y) = C_1 and V(x, y) = C_2, then the solution of given Lagrange's equation is $\phi(u, v) = 0$.

26. Solve : $(D^4 - D'^4)z = 0$

Put D = m, $D' = 1$

The auxillary equation is $m^4 - 1 = 0$

$$(m^2 + 1)(m^2 - 1) = 0$$

$$m = \pm i, \ \pm 1$$

$\therefore$ The solution is $z = f_1(y + ix) + f_2(y - ix) + f_3(y + x) + f_4(y - x)$ where f_1, f_2, f_3, f_4 are arbitrary functions.

27. Define homogeneous and non-homogeneous partial differential equations.

If the sum of degree of D and D^1 in each and every term of a PDE is equal, it is called homogeneous partial differential equation. Otherwise, it is called non-homogeneous partial differential equation.

28. **Form the PDE by eliminating the arbitrary function from** $z = f(x^2 - y^2)$**.**

(May-011, 2012)

$z = f(x^2 - y^2)$

$$p = \frac{\partial z}{dx} = 2xf' \Rightarrow \frac{p}{2x} = f'$$

$$q = \frac{\partial z}{dy} = 2yf' \Rightarrow \frac{q}{2y} = f'$$

$$\therefore \frac{p}{2x} = \frac{q}{2y}$$

The required PDE is $py - qx = 0$

29. Solve $yq + xp = z$ **(Nov-2010/2014)**

The given PDE is of the form Pp + Qq = R

Where P = x ; Q = y ; R = z

Lagrange's subsidiary equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

(i.e.) $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ …. (1)

Taking first two ratios in (1), we get

$$\frac{dx}{x} = \frac{dy}{y} \Rightarrow \int \frac{dx}{x} = \int \frac{dy}{y}$$

(i.e.) $\log x - \log y = \log C_1$

$$\frac{x}{y} = C_1$$

Taking second and third ratio, we get, $\frac{dy}{y} = \frac{dz}{z} \Rightarrow \int \frac{1}{y} dy = \int \frac{1}{z} dz$

(i.e.) $\log y - \log z = \log C_2$

$$\frac{y}{z} = C_2$$

Hence the general solution is $\boxed{\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0}$ where ϕ is arbitrary.

30. Find the complete solution of $p^2x^4 + y^2zq = 2z^2$ **(Nov-2010)**

$$p^2x^4z^{-2} + y^2zqz^{-2} = 2$$

$$\left(px^2z^{-1}\right)^2 + y^2qz^{-1} = 2 \qquad \ldots(1)$$

Let $X = \frac{1}{x}, Y = \frac{1}{y}, Z = \log z \Rightarrow \frac{\partial X}{\partial x} = \frac{-1}{x^2}, \frac{\partial Y}{\partial y} = \frac{-1}{y^2}, \frac{\partial Z}{\partial z} = \frac{1}{z}$

$$P = \frac{\partial Z}{\partial X} = \frac{\partial Z}{\partial z}\frac{\partial z}{\partial x}\frac{\partial x}{\partial X} = -x^2 pz^{-1}$$

$$Q = \frac{\partial Z}{\partial Y} = \frac{\partial Z}{\partial z}\frac{\partial z}{\partial y}\frac{\partial y}{\partial Y} = -y^2 qz^{-1}$$

Therefore (1) becomes

$$(-P)^2 + (-Q) = 2 \qquad \ldots(2)$$

$$P^2 - Q = 2$$

$\therefore$ Z=aX+bY+c where $a^2 - b = 2$ is the complete integaral of z.

$\therefore \log z = \frac{a}{x} + (a^2 - 2)\frac{1}{y}c$ is the complete solution.

31. **Form partial differential equation by eliminating the arbitrary constants from $z = (xa+by+a^2 + b^2)$.** **(May-2011)**

Given that $z = (xa+by+a^2 + b^2)$ … (1)

$\frac{\partial z}{\partial x} = p = a$ …(2)

$\frac{\partial z}{\partial y} = q = b$ …(3)

Substitute (2) & (3) in (1) we get

$z = px+qy+p^2+q^2$

$z = px+qy+p^2+q^2$ which gives the required partial differential equation.

32. **Write the subsidiary equation for Lagrange's linear equation.** **(May-2011)**

The subsidiary equation for Lagrange's linear Equation Pp + Qq = R is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

33. **Solve : $(D^3-3D^2D' + 2DD'^2)\, z = 0$** **(May-2011)**

Put D = m, $D^1 = 1$

The auxillary equation is $m^3 - 3m^2 + 2m = 0$

$m(m^2 - 3m + 2) = 0$

m = 0 is a root

The other roots are given by $m^2 - 3m + 2 = 0$

$(m - 2)(m - 1) = 0$

$m = 2, 1$

Therefore the roots are m= 0,1,2

$\therefore$ The solution is $\boxed{z = f_1(y + 2x) + f_2(y + x)} + f_3(y)$ where f_1, f_2 and f_3 are arbitrary functions.

34. **Solve :** $(D^2 - 2DD' + D'^2) = 0$ **(May-2011)**

Put D = m, $D^1 = 1$.

The auxillary equation is $m^2 - 2m + 1 = 0$

$(m - 1)^2 = 0$

$m = 1, 1$

$\therefore$ The solution is $\boxed{z = f_1(y + x) + x f_2(y + x)}$ where f_1, f_2 are arbitrary functions.

35. From the partial differential equation by eliminating the arbitrary constant $z = (x+a)(y+b)$

(Apr/May 2014)

Sol: Given $z = (x+a)(y+b)$ .....(1)

Differentiating partially (1) w.r.t x and y respectively we get

$$\frac{\partial z}{\partial x} = p = y + b$$

$$\frac{\partial z}{\partial y} = q = x + a$$

Substituting in (1) we get $z = pq$

36. Solve : $(D^2 - 6DD' + 9D'^2)z = 0$ **(Nov 2014)**

Sol: A.E is $m^2 - 6m + 9 = 0$

$(m-3)(m-3) = 0$

$\therefore m = 3, 3$

C.F = $f_1(y+3x) + xf_2(y+3x)$ where $f_1 and f_2$ are arbitrary functions.

37. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from

$(x-a)^2+(y-b)^2=z^2\cot^2\alpha$ **(Apr/May 2013)**

Sol: $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$ ……..(1)

Diff Partially (1) w.r.to x

$$2(x-a)=2z\cot^2\alpha\frac{\partial z}{\partial x}$$

$$(x-a)=zp\cot^2\alpha$$

Diff Partially (1) w.r.to y

$$2(y-b)=2z\cot^2\alpha\frac{\partial z}{\partial y}$$

$$(y-b)=zq\cot^2\alpha$$

Eqn (1) becomes

$$\left(zp\cot^2\alpha\right)^2+\left(zq\cot^2\alpha\right)^2=z^2\cot^2\alpha$$

$$z^2p^2\cot^4\alpha+z^2q^2\cot^4\alpha=z^2\cot^2\alpha$$

$$p^2\cot^2\alpha+q^2\cot^2\alpha=1$$

$$p^2+q^2=\tan^2\alpha$$

which is the required P.D.E

38. Eliminate the arbitrary constants a and b $z=ax+by+ab$**.** **(Nov 2013)**

Solution:

Given $z=ax+by+ab$ …… (1)

Diff Partially (1) w.r.to x and y ,

p=a and q=b

$z=px+qy+pq$, which is the required P.D.E.

39. Find the particular integral of $\left(D^2+4DD'\right)z=e^x$ **(Nov 2013)**

Solution:

$$\text{P.I}=\frac{1}{D^2+4DD'}e^x$$

$$=\frac{1}{1}e^x \qquad \text{Replace D by 1 and D' by 0}$$

$$=e^x$$

UNIT – II

APPLICATION OF PARTIAL DIFFERENTIAL EQUATIONS

BOUNDARY VALUE PROBLEMS

TRANSVERSE VIBRATION OF A STRETCHED STRING

1. **What is a boundary value problem? (May-2011)**

   Differential equations or partial differential equations that satisfy certain given conditions are called boundary conditions. Any differential equation together with these boundary conditions is called boundary value problem.

2. **State the assumptions made in the derivatives of one dimensional wave equation. (Nov 2013)**

   Solution:

   1. The motion takes place entirely in one plane.
   2. The effect of the friction is negligible
   3. The string is perfectly flexible
   4. TensionT in the stringto be so large that gravity may be negelected in comparison with it.

3. **State any four empirical laws when deriving the wave equation. (Nov-2010)**

   (i) The tension T caused by stretching the string before fixing it at the end points is constant.

   (ii) T is so large that other external forces such as weight of the string and friction may be considered negligible.

   (iii) The string is homogeneous and perfectly elastic.

   (iv) Displacement y and the slope $\frac{dy}{dx}$ at every point of a string are small.

4. **Write one dimensional wave equation (or) what is the constant a^2 in one dimensional wave equation? (Nov-2010,May-2012)**

   $$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad \text{where } a^2 = \frac{T}{m} = \frac{Tension}{mass}$$

5. **Write all possible solutions of wave equation (or) Write all possible solutions of the transverse vibrations of a string. (May, Nov-2010, 2012/Apr 2013)**

(i) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

(ii) $y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos pat + c_8 \sin pat)$

(iii) $y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$ are the possible solutions of the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

6. **Write 'D' Alembert's solution of the one dimensional wave equation**

$y(x,t) = \phi(x - at) + \psi(x + at)$ where ϕ and ψ are arbitrary functions. This is called general solution or D Alembert's solution of one dimensional wave equation.

7. **Write the general solution for displacement of a string of length ℓ whose end points are fixed and which starts from rest. (May-2011)**

The general solution is given by $y(x,t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)$

8. **Write the boundary and initial conditions for the transverse vibration of a string length ℓ with fixed ends with initial displacement f(x).**

The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

1. y (0, t) = 0 for every t $\geq$ 0
2. y (l, t) = 0 for every t $\geq$0
3. $\frac{\partial}{\partial t} y(x,t) = 0$ at t = 0 and 0 $\leq$ x $\leq l$
4. y(x, 0) = f(x)

9. Write the boundary conditions for the transverse vibration of a string of length *l* which is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = V_0 \sin^3\left(\frac{\pi x}{l}\right)$, $0 < x < l$ **.**

The one dimensional wave equation is given by $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

(i) y (0, t) = 0 $\forall$ t $\geq$ 0

(ii) y (*l*, t) = 0 $\forall$ t $\geq$ 0

(iii) y (x, 0) = 0, 0 < x <*l*

(iv) $\frac{\partial}{\partial t} y(x,t) = V_0 \sin^3\left(\frac{\pi x}{l}\right)$, o < x <*l* at t = 0.

10. Write the non-periodic solution of a one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{a^2 \partial^2 y}{\partial x^2}$

$u(x,t) = (A_1 e^{px} + A_2 e^{-px})(A_3 e^{pat} + A_4 e^{-pat})$

11. Solve the equation $2x\frac{\partial z}{\partial x} = -3y\frac{\partial z}{\partial y} = 0$ **by method of separation of variables.**

Given that $2x\frac{\partial z}{\partial x} = -3y\frac{\partial z}{\partial y} = 0$ …(1)

Let Z = X(x)Y(y) be the solution of (1)

$\frac{\partial z}{\partial x} = X'Y; \frac{\partial z}{\partial y} = XY'$

Substituting in (1), we get

2x X'Y – 3y XY' = 0

2x X'Y = 3y XY'

$2x\frac{X'}{X} = 3y\frac{Y'}{Y}$

Let $2x\frac{X'}{X} = 3y\frac{Y'}{Y} = k$

Consider $2x\frac{X'}{X}=k$

$$2\frac{X'}{X}=\frac{k}{x}$$

integrating w.r.t. 'x' we get

$$2\int\frac{X'}{X}dx=k\int\frac{1}{x}dx$$

$$2\log X=k\log x+\log A$$

$$\log X^2-\log x^k=\log A$$

$$\log\left(\frac{X^2}{x^k}\right)=\log A$$

$$\frac{X^2}{x^k}=A\Rightarrow X=\sqrt{Ax^k}=ax^{k/2} \qquad \ldots(2)$$

Consider $3y\frac{Y'}{Y}=k$

$$\frac{3Y'}{Y}=\frac{k}{y}$$

Integrating both sides, we get

3 log Y = k log y + log B

log Y^3 = log y^k + log B

log Y^3 = log By^k

$$Y^3=By^k\Rightarrow Y=by^{k/3} \qquad \ldots(3)$$

Substituting (2) & (3) in Z, we get

$$Z=ax^{k/2}by^{k/3}$$

$$Z=abx^{k/2}y^{k/3}$$

is the required solution.

12. Write the general solution for displacement of a string of length *l* whose end points are fixed with non-zero initial velocity

$$y(x,t)=\sum_{n=1}^{\infty}c_n\sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right)$$

13. Solve the equation $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ **(Nov-2012)**

Given that $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ …(1)

Let Z = X(x)Y(y) be the solution of (1)

$\frac{\partial z}{\partial x}=X'Y;\frac{\partial z}{\partial y}=XY'$

Substituting in (1), we get

3X'Y +2 XY' = 0

3 X'Y = -2 XY'

$3\frac{X'}{X}=-2\frac{Y'}{Y}$

Let $3\frac{X'}{X}=-2\frac{Y'}{Y}=k$

Consider $3\frac{X'}{X}=k$

$$3\frac{X'}{X}=k$$

integrating w.r.t. 'x' we get

$3\int\frac{X'}{X}dx=k\int dx$

$3\log X=kx+A$

$\log X^3=kx+A$

$X^3=e^{kx+A}$

$X=e^{\frac{kx+A}{3}}=ae^{\frac{kx}{3}}$ …(2)

Consider $-2\frac{Y'}{Y}=k$

Integrating both sides, we get

-2log Y = ky + B

log Y^{-2} = ky + B

$\log Y^{-2}=ky+B$

$$\frac{1}{y^2} = e^{ky+B}$$

$$y^2 = e^{-ky-B}$$

$$y = e^{\frac{-ky-B}{2}}$$

$$y = be^{\frac{-ky}{2}}$$

$$\ldots(3)$$

Substituting (2) & (3) in Z, we get

$$Z = abe^{kx/3}e^{-ky/2}$$

$$Z = abe^{kx/3}e^{-ky/2}$$

$$Z = abe^{kx/3 - ky/2}$$

is the required solution.

14. Classify the partial differential equation $u_{xx} + xu_{yy} = 0$ (Nov 2014)

Solution:

Here A = 1 , B = 0 , C = x

$\therefore B^2 - 4AC = -4x$

If x=0 Þit is Parabolic

If x<0 Þit is Hyperbolic

If x>0 Þit is Elliptic

15. Write the most suitable solution of the one dimensional wave equation (Apr/May 2014)

Solution:

The suitable solution of the one dimensional wave equationis

$$y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos pat + c_8 \sin pat)$$

16**. A string is stretched and fastened at two pints x=0 and x=l at a distance 'l' apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time t=0 . Write down the boundary conditions in mathematical form. (Apr/May 2014)**

Solution:

The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

17. **Explain the method of separation of variables.** (NOV 2014)

Let Z be the dependent variable and (x,y) be the independent variable. Assume the solution to be the product of two functions, one of them a function of x alone and the other function of y alone. Then the solution of P.D.E is converted into the solution of O.D.E.

18. **Explain initial value problem.**

Solution:

19. State the wave equation in Cartesian form.

Solution:

The solution of one dimensional wave equation in Cartesian form is given by

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

Here y is a function of x and t.

UNIT III

ONE DIMENSIONAL AND TWO DIMENSIONAL HEAT FLOW EQUATIONS

1. **Write one dimensional heat flow equation.**

   The one dimensional heat flow equation is given by $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ where $\alpha^2 = \frac{k}{c\rho}$ is called diffusivity of the material of the body through which heat flows. Here ρ - density, c-specific heat, k- thermal conductivity.

2. **Give three possible solutions of one dimensional heat flow equation.**

   The one dimensional heat flow equation is given by $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

   The three possible solutions are

   (i) $u(x,t) = e^{\alpha^2 p^2 t}(A_1 e^{px} + A_2 e^{-px})$

(ii) $u(x,t) = e^{-\alpha^2 p^2 t}(A_3 \cos px + A_4 \sin px)$

(iii) $u(x,t) = A_5 x + A_6$

3. Write the correct solution of one dimensional heat flow equation, $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

The correct solution is given by $u(x,t) = e^{-\alpha^2 p^2 t}(A\cos px + B\sin px)$

4. What are the assumptions made while deriving one dimensional heat flow equation or diffusion equation? (May-2012)

(i) Heat flows from higher to lower temperature.

(ii) The quantity of heat required to produce a given temperature change in a body is proportional to the mass of the body and the temperature change.

(iii) The rate at which heat flows across any area is jointly proportional to the area and to the temperature gradient normal to the area. This is known as Fourier's law of heat conduction.

5. Write one dimensional diffusion equation.

The one dimensional diffusion equation is given by $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ where α^2 is called diffusivity of the material of the body.

6. Write the initial and boundary conditions of one dimensional heat flow equation.

The one dimensional heat flow equation is $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

The boundary conditions are

(i) $u(o,t) = k_1^0, \forall t \geq 0$

(ii) $u(o,t) = k_2^0, \forall t \geq 0$ [l being the length of the 1-dimensional rod]

(iii) The initial condition is u(x, 0) = f(x), 0 < x <l.

7. Distinguish between steady and unsteady states in heat conduction problems.(Nov 2014)

Steady State	Unsteady state
The temperature at any	The temperature at any point of the

point depends only on the position of the point and is independent of the time 't'	body depends on the position and time 't'

8. Define solution for one dimensional heat flow equation in steady state condition.

(NOV 2013/2014/APR 2014)

The one dimensional heat flow equation is $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ …(1)

In steady state condition, the temperature 'u' depends only on x and not time 't'.

Hence $\frac{\partial u}{\partial t} = 0$

$\therefore$ (1) becomes $\frac{d^2 u}{dx^2} = 0$

$\Rightarrow u = ax + b$ where a, b are arbitrary constants.

9. Write down the Laplace equation in two dimension (May-2011, Nov-2010)

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ [This is known as Laplace's equation]

10. Write the different solutions of Lap lace's equation in Cartesian co-ordinates.

(i) $u(x, y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

(ii) $u(x, y) = (c_5 \cos px + c_6 \sin px)(c_7 e^{py} + c_8 e^{-py})$

(iii) $u(x, y) = (c_9 x + c_{10})(c_{11} y + c_{12})$

11. Write down the equation of heat flow in two dimension in polar co-ordinates. [Polar form of Laplace equation] (Nov-2010, May- 2012)

$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

12. Write all the solutions of Laplace equation in polar co-ordinates.

(Nov-2010, May-2012)

(iii) $u(r,\theta) = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta)$

(iv) $u(r,\theta) = (c_5 \cos(p \log r) + c_6 \sin(p \log r))(c_7{}^{p\theta} + c_8{}^{-p\theta})$

(v) $u(r,\theta) = (c_9 \log r + c_{10})(c_{11}\theta + c_{12})$

13. For $r^2 u_{rr} + r u_r + u_{\theta\theta}$, write a solution which is periodic in θ.

$u(r,\theta) = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta)$.

14. A semi circular plate of radius 'a' cms has its temperature at 0^0C on the bounding diameter and 100^0C on its circumference. Write the corresponding boundary conditions.

Let u(r, θ) be the temperature at any point (r, θ) in the steady state. The boundary conditions are

(i) u (r, θ) = 0, $0 \le r \le a$

(ii) u(r, π) = 0, $0 \le r \le a$

(iii) u(a, θ) = 100, $0 \le 0 \le \pi$

15. Write two dimensional heat flow equation in transient state. (Nov-2010)

The Transient state equation of two dimensional heat flow is $\frac{\partial u}{\partial t} = \alpha^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right]$

16. Write a solution which is periodic in y for $U_{xx} + U_{yy} = 0$.

$u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

17. **Explain the term"thermally insulated ends". (Nov 2013)**

If an end of a heat conducting body is thermally insulated, it means that no heat passes through that section. Mathematically the temperature gradient is zero at that point (i.e.) $\frac{\partial u}{\partial x} = 0$

18. Write down the form of general solution of one dimensional heat flow equation, when both the ends of the rod are insulated?

The 1-dimensional heat flow equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

With boundary conditions (i) $u(0,t) = 0, \forall t \geq 0$

(ii) $u(l,t) = 0, \forall t \geq 0$

(iii) $u(x,0) = f(x), 0 < x < l$

has the most general solution $u(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} e^{-n^2\pi^2\alpha^2 t/l^2}$

19. How do you choose the proper solution of two dimensional heat flow equation in steady state condition? (May-2011)

(i) If the non-zero boundary value is prescribed on x = 0 or x = a, the correct solution is $u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$

(ii) If the non-zero boundary value is prescribed on y = 0 or y = b, the correct solution is $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

20. Write any two solutions of the two dimensional heat flow equation under steady state conditions in Cartesian coordinate system. (Nov-2012)

If the non-zero boundary value is prescribed on x = 0 or x = a, the correct solution is $u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$

If the non-zero boundary value is prescribed on y = 0 or y = b, the correct solution is $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

21. A rod 30cm long has its ends A and B kept at 20° C and 80° C respectively until steady state condition prevails. Find the steady state temperature in the rod. (Nov-2012)

In steady state condition, the heat flow is given by $u(x) = Ax + B$ …(1)

Given that, u(0)=20

u(30)=80

On substituting this in equation (1), we get

u(0)=A(0)+B=20 $\Rightarrow$ B=20

u(30)=A(30)+20=80

A(30)=60

A=2

$\therefore u(x) = 2x + 20$

22. A rod 10cm long has its ends A and B kept at $20^{\bullet}$ C and $70^{\bullet}$ C respectively until steady state condition prevails. Find the steady state temperature in the rod.

(May-2011)

In steady state condition, the heat flow is given by $u(x) = Ax + B$ $\quad \ldots(1)$

Given that, u(0)=20

u(10)=70

On substituting this in equation (1), we get

u(0)=A(0)+B=20 $\Rightarrow$ B=20

u(10)=A(10)+20=70

A(10)=50

A=5

$\therefore u(x) = 5x + 20$

24. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation. (Nov 2014)

Sol: In one dimensional wave equation the suitable solution contain trigonometric terms as y must be periodic function of x and t $u(x,y) = (A\cos px + B\sin px)(c\cos pt + D\sin pt)$

In one dimensional heat equation, since $u(x,t)$ must decrease with increase of time therefore the solution suitable in this case is $u(x,t) = (A\cos px + B\sin px)e^{-c^2p^2t}$

26. Write the possible solutions of the 2 dimensional heat equation in Cartesian coordinate.

(Apr/May 2014)

Solution:

The possible solution of two dimensional heat equation in Cartesian coordinate is given by

$(i)\ u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$

$(ii)\ u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

$(iii).\ u(x,y) = (Ax + B) + (Cx + D)$

27. When will you use the non-zero boundary conditions in one dimensional heat equation?

Solution:

During the unsteady state condition we will use the non-zero boundary conditions in one dimensional heat equation.

28. Write down the governing equation of two dimensional steady state heat condition.

Solution:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

29. State any two laws under which Laplace transform exists. (Nov 2004)

Solution:

(i). The sides of the bar are insulated so that the loss or gain of heat from the sides by conduction or radiation is negligible.

(ii). The same amount of heat is applied at all points of the face.

30. State Fourier law of heat conduction.

Solution:

The rate at which heat flows across any area is proportional to the area and to the temperature gradient normal to the curve. This constant of proportionality is known as Thermal conductivity of the material. It is known as Fourier law of heat conduction.

UNIT – IV
SAMPLING THEORY
LARGE SAMPLES

1. **What is Population? (Nov 2014)**

   The group of individuals which may be persons or experimental outcomes under study is called population or universe. It may be finite or infinite.

2. **What is a sample?**

   A sample is a finite subset of the population that is studied to learn about the characteristics of the population.

3. **Write the different types of sampling.**

   (i) Purposive sampling

   (ii) Random sampling

   (iii) Stratified sampling

   (iv) Systematic sampling

4. **Define sampling distribution.**

Consider all possible samples of size 'n' drawn from a given population at random. We calculate mean values of these samples. If we group these different means according to their frequencies, the frequency distribution so formed is called sampling distribution.

5. **Define parameter and statistic.**

The statistical constants such as mean (μ), variance (σ^2) etc of the population is called as parameters and the statistical measures computed from sample such as mean ($\bar{x}$), variance (s^2), etc are called as statistic.

6. **Define standard error.** **(May-2011)**

The standard deviation of the sampling distribution of a statistic is known as its standard error, abbreviated as S.E

7. **Define test of significance.**

The procedure which enables us to decide whether to accept or reject the hypothesis is called the test of significance.

8. **Write the procedure for testing a hypothesis.**

1. Formulate H_0 and H_1
2. Choose level of significance α
3. Compute test statistic Z, using the data available in the problem.
4. Pick out the critical value at α% level say Z_α
5. Draw conclusion

   If $|Z| < Z_\alpha$, accept H_0 at α% level

   Otherwise, reject H_0 at α% level.

9. **Define null and alternate hypothesis.** **(Nov-2010,May2012/2014)**

Null hypothesis

It is a definite statement about the population parameter which is usually a hypothesis of no difference and is denoted by H_0.

In case of signal statistic, H_0 will be that the sample statistic does not differ significantly from the hypothetical parameter value.

In case of two statistics, H_0 will be that the sample statistics do not differ significantly.

Alternative hypothesis

Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, usually denoted by H_1

10. **Explain type I and type II errors (or) Explain producer's risk and consumer's risk.** **(May 2011)**

Type I error : Reject H_0 when it is true

Type II error : Accept H_0 when it is wrong.

11. Define level of significance. (May 2011)

Level of significance of a test denoted by α is the probability of committing type I error. Thus, it measures the amount of error associated in taking decisions.

(ie) If α = 0.05 = 5 %, then one is 95 % confidednt that a right decision is made. It is also known as size of the test.

12. Define critical region.

The critical region of a test of statistical hypothesis is the region of the normal curve which corresponds to the rejection of null hypothesis.

13. Define critical value (or) significant value.

It is the value of a test statistic which divides the area under the probability curve into critical and non-critical region.

14. Explain one tailed and two tailed test. (Apr 2014)

When only one tail of the sampling distribution of the normal curve is used, the test is called one-tailed test. One tailed test willbe either left tailed or right tailed test. When two tails of the sampling distribution of the normal curve are used, the relevant test is called two-tailed test.

15. Define confidence interval.

The interval for which one can be 100% confident that the parameter under investigation lies in this interval is called confidence interval. The two end points of the confidence interval are known as confidence limits.

16. Define degrees of freedom.

Degrees of freedom of a statistic is a positive integer, denoted by v = n – k when 'n' is the number of independent observations of the random sample and k is the number of population parameters calculated using sample data.

17. Write 95% and 99% confidence limits for population mean μ.

For 95%, $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$

For 99%, $\bar{x} - 2.58\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.58\frac{\sigma}{\sqrt{n}}$

18. Write the fiducial limits of population proportion P at α level of significance.

$$p \pm Z_\alpha \sqrt{\frac{pq}{n}}$$

In particular, 95% confidence limits for P are given by $p \pm 1.96\sqrt{\frac{pq}{n}}$

For 99%, $p \pm 2.58\sqrt{\frac{pq}{n}}$

19. Write test statistic for single proportion.

$$Z = \frac{p - P}{\sqrt{PQ/n}} \sim N(0, 1)$$

Where p – sample proportion

P – Population proportion

Q = 1 – P

n – sample size

20. Write test statistic for difference of proportion.

$$Z = \frac{p_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

21. Write test statistic for single mean.

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$\bar{x}$ – sample mean

μ - population mean

σ – population standard deviation

n – sample size

22. Write test statistic for difference of means.

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\bar{x}_1, \bar{x}_2$ – sample means

σ_1, σ_2 – population standard deviation

n_1, n_2 – sample size

23. Write test statistic for the difference of S.D.

$$Z = \frac{S_1 - S_2}{\sqrt{\frac{S_1^2}{2n_1} + \frac{S_2^2}{2n_2}}}$$

S_1, S_2 – sample S.D

n_1, n_2 – sample size

24. Critical values for one and two – tailed tests.

Critical value	**Level of significance (α)**		
	1%	**5%**	**10%**
Two tailed	2.58	1.96	1.645
Right tailed	2.33	1.645	1.28
Left tailed	-2.33	-1.645	-1.28

25. State some standard approximating curves.

(i) $y = a_0 + a_1x$ – straight line

(ii) $y = a_0 + a_1x + a_2x^2$ - parabola or quadratic curve

(iii) $y = AB^x$ - exponential curve

(iv) $y = AX^B$ – Geometric curve.

26. Write normal equations for $y = a_0 + a_1X$.

$\sum y = na_0 + a_1\sum x$

$\sum xy = a_0\sum x + a_1\sum x^2$

27. Write normal equations for $y = a_0 + a_1x + a_2x^2$. **(May 2011)**

$\sum y = na_0 + a_1\sum x + a_2\sum x^2$

$\sum xy = a_0\sum x + a_1\sum x^2 + a_2\sum x^3$

$\sum x^2y = a_0\sum x^2 + a_1\sum x^3 + a_2\sum x^4$

28. Write normal equations for $y = AB^x$.

$Y = AB^x$

Takme log on both sides

$\log y = \log A + x\log B$

$Y^* = A^* + XB^*$ Where $Y^* = \log Y$, $A^* = \log A$, $B^* = \log B$

The normal equations are

$\sum Y^* = nA^* + B^* \sum x$

$\sum xy^* = A^*\sum x + B^*\sum x^2$

29. Write normal equations for $y = ax^b$.

$Y = a\,x^{\,b}$

Take log on both sides

Log y = log a + b log x

$Y^* = A^* + bx^*$ where $Y^* = \log Y$, $A^* = \log a$, $X^* = \log x$

The normal equations are

$\sum y^* = nA^* + b\sum X^*$

$\sum X^*Y^* = A^*\sum x^* + b\sum (x^*)^2$

30. What is the general form of a test statistic.

$$Z = \frac{t - E(t)}{S.E(t)}$$

31. What is the S.E. of the difference between the means of two large samples?

(i) $S.E(\overline{x_1} - \overline{x_2}) = \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ if $\sigma_1 = \sigma_2 = \sigma$

(ii) $S.E(\overline{x_1} - \overline{x_2}) = \sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}$ if sample S.D. s_1& s_2 are only given.

32. What is the S.E. of the difference between the S.D's of two large samples

(i) $S.E(s_1 - s_2) = \sigma\sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}$

(ii) $S.E(s_1 - s_2) = \sqrt{\frac{s_1^{\,2}}{2n_1} + \frac{s_2^{\,2}}{2n_1}}$ if σ is not known.

33. A coin is tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased. (Nov 2012)

Given n=400, $p = \frac{216}{400} = 0.54$

P = Probability of getting head= ½=0.5

Q = ½ = 0.5

H_0: P = 0.5, the coin is unbiased.

H_1: P ≠ 0.5, the coin is biased.

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54-0.5}{\sqrt{\frac{0.5X0.5}{400}}} = 1.6$$

The table value at5% level of significance=1.96

Z = 1.6 < 1.96

(i.e.) Calculated value is less than Table value.

∴ H0 is accepted.

∴ The coin is unbiased

34. Write down the principle of least squares. (Nov 2012/Apr 2013)

Let (x_i,y_i) ,i=1,2,…n be n sets of observations and let y = f(x). Let the residual $d_i = y_i - f(x_i)$, E= $\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n}(x_i - f(x_i))^2$ is the sum of the squares of the residuals. If E=0, then all the n points will lie on y = f(x). Otherwise, we choose f(x) such that E is minimum. This method is called method of least squares.

35. A sample of 900 members from a normal population with S. D. 2.61cms has a mean 3.5cms. Find the 95% fiducial limits for the population mean. (Nov-2010)

Given that n=900, σ =2.61, $\bar{x}$ =3.5

95% fiducial limits for the population mean, $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$

$$3.3295 \le \mu \le 3.6705$$

36. A sample of 900 itemshas the mean 3.4 and S. D. 2.61. Can the sample be regarded as drawn from a population with mean 3.25 at 5% level of significance (May-2011)

Given that n=900, σ =2.61, $\bar{x}$ =3.4, μ=3.25

H_0: μ=3.25

H_1 : $\mu \neq 3.25$

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{3.4-3.25}{2.61/\sqrt{900}} = \frac{0.15}{2.61/30} = 1.724$$

$\bar{x}$ – sample mean

μ - population mean

σ – population standard deviation

n – sample size

Calculated value 1.724<table value $t_{0.05}$ (1.96)

$\therefore H_0$ is accepted.

37. **Define large Sample & Small Sample.(Nov 2013/2014)**

If n < 30, the sample is called small sample and if n $\geq$ 30, it is called large sample.

39. What is the formula for finding difference between the sample proportion and Population proportion. (Apr 2013)

To test the significance difference between

(i) Two sample proportions

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{Where } P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \text{ and } Q = 1 - P$$

$$Z = \frac{P_1 - P_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}}$$ When sample proportions are not given.

(ii) Two population proportion

$$Z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

UNIT – V

SMALL SAMPLES

1. **Define samples size.**

   The number of items selected ina sample is called the sample size and is usually denoted by n. If n < 30, the sample is called small sample and if n $\geq$ 30, it is called large sample

2. **Define student's t-test for single mean.**

   The student's t-test for single mean is defined as

$$t = \frac{|\bar{x} - \mu|}{s/\sqrt{n}} \quad \text{where S} = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$$

$\bar{x}$ - Sample mean

μ - Population mean

n – Sample size

Number of degrees of freedom = n -1.

3. What are the assumptions of student's t-test? (May-2011)

(i) The parent population from which the sample drawn is normal.

(ii) The sample observations are independent. (i.e.) the sample is random.

(iii) The population S.D. σ is unknown.

4. Define student's t-test for difference of means.

Student's t-test for difference of means is defined as

$t = \dfrac{\left|\bar{x}_1 - \bar{x}_2\right|}{S\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ where $\bar{x}_1$, $\bar{x}_2$ – sample means , n_1, n_2 – sample sizes&

$$S^2 = \frac{n_1 s_1^{\,2} + n_2 s_2^{\,2}}{n_1 + n_2 - 2} \text{ (or) } S^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

The number of degrees of freedom = $n_1 + n_2 - 2$

5. Write two applications of t-test.

It is used

(i) To test the significance of the difference between two sample means

(ii) To test the significance of the coefficient of correlation.

6. Define t-test for paired observations. (May-2012)

When two samples are of the same size and the data are paired, the statistic is defined

as $t = \dfrac{\bar{d}}{s/\sqrt{N}}$ where S = $\sqrt{\dfrac{\sum (d - \bar{d})^2}{N - 1}}$ with D.O.f = N -1

$\bar{d}$ - mean of differences

N - sample size

7. Write 95% confidence limits for the population mean μ in a small sample test.

$\bar{x} \pm t_\alpha \dfrac{S}{\sqrt{n}}$ where $S^2 = \dfrac{\sum (x - \bar{x})^2}{n - 1}$

n – Sample size

8. Define F-test for equality of two population variances.

To test whether

(i) Two independent samples have been drawn from the normal populations with the same variance σ^2 (or)

(ii) Two independent estimates of the population variances are homogeneous or not;

F – test is defined as $F = \frac{S_X^2}{S_Y^2}$ where $S_X^2 > S_Y^2$

and $$S_X^2 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_i - \bar{x})^2$$

$$S_Y^2 = \frac{1}{n_2 - 1}\sum_{i=1}^{n_2}(y_i - \bar{y})^2$$ with d.o.f. (n_1-1, n_2-1)

9. Define ψ^2 – test for goodness of fit. (May-2011, 2012)

If O_i (i = 1, 2, ... n) is a set of observed (experimental) frequencies and E_i (i = 1, 2, . . . n) is the corresponding set of expected (theoretical) frequencies, then the statistic ψ^2 is defined as $\psi^2 = \sum_{i=1}^{n}\frac{(O_i - E_i)^2}{E_i}$ with d.o.f = n - 1

10. What are the conditions for applying chi-square test?

For the validity of chi-square test of "goodness of fit" between theory and experiment, the following conditions must be satisfied.

(i) The sample observations must be independent.

(ii) Constraints on the cell frequencies, if any, should be linear.

(iii) N, the total frequency should be reasonably large, say greater than 50.

(iv) No theoretical cell frequency should be less than 5.

11. Write any two uses of ψ^2 test. (or) what are the applications of ψ^2 –test?

(Nov-2010, Nov-2012/2014)

ψ^2 – test is used to test

(i) The independence of attributes.

(ii) The goodness of fit.

(iii) Whether two or more attributes are associated or not.

(iv).Whether the theoretical distributions such as Binomial, Poisson, Normal etc., fit empirical distributions.

12. Write procedure of ψ^2 –test for goodness of fit.

(i) H_0: Good-fit exists between theoretical distribution and given data (observed frequency)

(ii) H_1 :No good fit.

(iii) Level of significance α : as prescribed

(iv) Compute τ^2 and choose degrees of freedom.

(v) If ψ^2 > tabulated value, reject H_0 (i.e.) poor fit

(iv) If ψ^2 < tabulated value, accept H_0 (i.e.) good fit.

13. Write the number of degrees of freedom for fitting binomial, Poisson and Normal distribution.

For Fitting Binomial distribution, v = n- 1

For Fitting Poisson distribution, v = n – 2

For Fitting Normal distribution,v = n – 3.

14. Define binomial distribution.

A discrete random variable X is said to follow binomial distribution if its probability mass function is given by

$$P(x = r) = nc_r\, p^r\, q^{n-r},\ r=0,1,2,\ldots n.$$

15. Define Poisson distribution.

A discrete random variable x is said to follow Poisson distribution if its probability mass function is given by $p(x = x) = \frac{e^{-\lambda}\lambda^x}{x!}$, x = 0, 1, 2, …∞

16. What is contingency table?

The matrix or tabular form of the sample data is called contingency table.

17. Write down the value of ψ^2 for a 2X2 contingency table with cell frequencies a, b, c and d. (May 2014)

2 x 2 contingency table with cell frequencies a, b, c, d is

a	b	a + b
c	d	c + d
a + c	b + d	**A + b + c + d = M**

$$\psi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$ where N = a + b + c + d.

18. What is Yates' correction for continuity of ψ^2 ?

When the approximate values of ψ^2 are discrete, Yates has shown that if

$$\psi^2 = \sum\left[\frac{\left(|o_i - E_i| - \frac{1}{2}\right)^2}{E_i}\right]$$ is used ψ^2 - approximation is improved. This is called Yates' correction.

Note: This can be used only for 2 x 2 contingency table, it is used only when some cell frequency is small (less than 5).

19. What is parametric test?

The tests that are based on the assumptions about the population parameters are called parametric test.

(e.g.) t – test, F – test, Z – test.

20. What is Non-parametric test?

The tests which are not based on the assumptions about population parameters are called as Non-parametric test. (e.g.) Chi-Square test.

21. A random sam[ple of size 16 has 53 as mean. The sum of the squares of the deviation frommean is 135. Can this be regarde as taken from the population having 56 as mean?

Given that n=16, sample mean $\bar{x} = 53$, $\sum(x-\bar{x})^2$ =135, μ =56

H_0: μ =56

H_1: $\mu \neq 56$

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{135}{15}} = 3$$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{53-56}{3/4} = -4$$

$|t|$ =4, Degrees of freedom=n-1=15

The table value=1.753

Calculated value $|t|$ > table value.

$\therefore$ H_0 is rejected.

22. Give any two properties of the t-distribution. (Nov2010/2014)

(i) The t-distribution is symmetric and bell shaped and the shape is dependent on the sample size.

(ii) Mean of the distribution is zero and the variance is greater than 1

23. Define Student's t-distribution with (n-1) degrees of freedom. (May-2011/2014)

In probability and statistics, Student's t-distribution (or simply the t-distribution) is a family of continuous probability distributions that arises when estimating the mean of a normally distributedpopulation in situations where the sample size is small and population standard deviation is unknown.

24. Define independence of attributes. (Nov 2013)

Two attributes A and B are said to be independent if there exists no relationship of any kind between them.

25. Write the uses of F- distribution.(Nov 2013/2014)

(i). It is used to find whether two independent estimates of population variance differs Significantly (or)

(ii). It is used to find whether two samples may be regarded as drawn from the normal Population having the same variance.

Question Bank

UNIT I

ARBITRARY CONSTANTS:

1. Form partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2+(y-b)^2+z^2=c^2$

ARBITRARY FUNCTIONS:

1. Form partial differential equation by eliminating the arbitrary function from $f(xy+z^2, x+y+z)=0$
2. Form partial differential equation by eliminating the arbitrary function from $\phi(x+y+z, x^2+y^2-z^2)=0$
3. From the partial differential equation by eliminating the arbitrary function f and ϕ from $z=f\left(x^2+y\right)+\varphi\left(x^2-y)\right)$
4. Solve: Form the PDE by eliminating arbitrary function from $z=f(2x+3y)+g(2x+y)$

STANDARD TYPES:

1. Solve: $p(1+q)=qz$
2. Solve: $z^2(p^2x^2+q^2)=1$
3. Solve: $p^2+pq=z^2$
4. Solve: $9(p^2z+q^2)=4$

CLARIUTS FORM:

5. Find the singular integral of $z=px+qx+\left(\frac{q}{p}-p\right)$
6. Find the complete and singular solution of $z=px+qy+p^2-q^2$
7. Solve: $z^2(p^2+q^2)=(x^2+y^2)$

LAGRANGE EQUATION:

1. Solve the equation $(z^2-2yz-y^2)p+(xy+zx)q=xy-zx$
2. Solve the equation $y^2p-xyq=x(z-2y)$
3. Solve: $x(y^2+z)p+y(x^2+z)q=z(x^2-y^2)$
4. Solve: $x(y^2+z^2)p+y(z^2+x^2)q=z(y^2-x^2)$

5. Solve: $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$

6. Solve $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \frac{x-y}{xy}$.

7. Solve: $(3z-4y)\frac{\partial z}{\partial x} + (4x-2z)\frac{\partial z}{\partial y} = (2y-3x)$

HOMOGENOUS EQUATION:

1. Solve: $(D^3 - 7DD^{1^2} - 6D^{1^3})z = e^{2x+y} + \sin(x+2y)$
2. Solve: $(2D^2 - 5DD^1 + 2D^{1^2})z = 5\sin(2x+y)$
3. Solve the equation $(D^2 + 4DD' - 5D'^2)z = xy + \sin(2x+3y)$
4. Solve $\left(D^2 - 2DD' + D'^2\right) z = x^2 y^2 e^{x+y}$
5. **Solve:** $(D^2 + 3DD' - 4D'^2)Z = \sin y$
6. Solve: $(D^2 - 2DD')z = e^{2x} + x^{3y}$
7. Solve: $(D^2 - D^{1^2})z = e^{x-y} + x^2 y$
8. Solve: $(D^2 + 4DD^1 - 5D^{1^2})z = 3e^{2x-y} + \sin(x-2y)$
9. Solve $\left(D^3 - 4D^2D' + 4DD'^2\right)\ z = 6\sin\left(3x+6y\right)$.

NON-HOMOGENOUS EQUATION:

1. Solve the equation $(D^2 - D'^2 - 3D + 3D')z = e^{x+2y} + xy$
2. Solve: $(D+1)(D+D^1-1)z = 5e^x$
3. Solve: $(D^2 + 2DD' + D'^2 - 2D - 2D^1)z = e^{x-y} + x^2 y$
4. Solve: $(D^3 - 2D\ D^1)z = 3x^2 y + \sin(x+2y)$
5. Solve: $(D^2 - 2DD' + D'^2 - 3D + 3D^1 + 2)z = e^{2x-y}$
6. Solve: $(D^2 + 2DD' + D'^2 + 2D + 2D^1 + 1)z = 0$

UNIT II

METHOD OF SEPRATION OF VARIABLES:

1. Solve by method of separation of variables $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ when $u(x,0) = 4e^{-x}$
2. Solve by method of separation of variables $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 3u$ where $u(0,t) = e^{-5t}$

3. Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, u(0, y) = 8e^{-3y}$ by using method of separation of variables
4. Solve by using method of separation of variables the equation $x^2 q + y^3 p = 0$.
5. Classify the following p.de

   (i) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ (one dimensional heat equation)

   (ii) $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ (one dimensional wave equation)

   (iii) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial y^2} = 0$ (one dimensional heat equation)
6. Find all the possible solutions of the PDE $y_{tt} = a^2 y_{xx}$ using separation of variables method.

<u>Zero Velocity</u>

1. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is vibrating, each point is given a velocity $\lambda x(l - x)$, find the displacement of the string at any point and at any time.
2. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $f(x) = \begin{cases} x, & 0 < x < l/2 \\ l - x & l/2 < x < l \end{cases}$. It is released from rest from this position, find the displacement of the string at any point x from one end and at any time t.
3. A string is stretched and fastended to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$ .Find the displacement of any point on the string at a distance of x from one end at time t.
4. A uniform string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form of the curve $y = K\sin^3\left(\frac{\pi x}{l}\right)$ and then releasing it form this position at time t=0 . Find the displacement of the point of the string at a distance 'x' form one end at time 't'.

<u>Bisection [Mid point]</u>

1. A string is tightly stretched and its ends are fastened at two points x = 0 and x = 2l. The midpoint of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string.

2. The points of trisection of a string of length *'2l'* are pulled aside through a distance *'d'* on opposite sides of the position of equilibrium and the string is released from rest. Write the mathematical form of this physical situation
3. A string is tightly stretched and its ends are fastened at two points $x = 0$ and $x = l$. The midpoint of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string.

<u>Tri Section</u>

1. The points of trisection of a string of length *'2l'* are pulled aside through a distance *'d'* on opposite sides of the position of equilibrium and the string is released from rest. Write the mathematical form of this physical situation

<u>Non – Zero Velocity</u>

1. If a string of length l is initially at rest in its equilibrium position and each of its points is given a velocity v such that $v = \begin{cases} cx & 0 < x < l/2 \\ c(l-x) & l/2 < x < l \end{cases}$ Find the displacement of the string at any time t.
2. A string is stretched between two fixed points at a distance of 60 cm and the points of the string are given initial velocities v, above $v = \begin{cases} \dfrac{\lambda x}{30}, & 0 < x < 30 \\ \dfrac{\lambda}{30}(60-x), & 30 < x < 60 \end{cases}$

   'x' being the distance from an end point. Find the disp0klacement of the string at any time.
3. A taught string of length 20cms fastened at both ends is displaced from its position of equilibrium, by imparting to each of its points an initial velocity given by

$$v = \begin{cases} x & 0 < x < 10 \\ 20-x, & 10 < x < 20 \end{cases}$$

x being the distance from one end. Determine the displacement of the string at any subsequent time.

4. A tightly stretched string with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $kx(l-x)$. Find the displacement y(x, t).

UNIT III

Ordinary Problems

1. Find the solution of the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the condition.

   $u(0,t) = 0, u(l,t) = 0 \; fort \geq 0 and \; u(x,0) = u_0 \; for \; all \; x$

2. Solve the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, subject to the boundary conditions

   $u(0,t) = 0, u(l,t) = 0, u(x,0) = x$

3. A long rectangular plate has its surfaces insulated and the 2 long sides as well as one of the short edges are maintained at $0^\circ c$. Find an expression for the steady state temperature u(x,y), the short edge y=0 is kept at $u_0{}^\circ c$.

Zero Boundary Conditions:

1. A uniform bar of length 'l' thorough which heat flows is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given $K(lx) - x^2$, for $0 < x < l$. Find the temperature distribution in the bar after time 't'.
2. A rod 30cm, long has its ends A and B kept at $40^\circ c$ and $100^\circ c$ respectively. Until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^\circ c$ and kept so. Find the temperature u(x, t).
3. A rod of length 1 has its ends A and B kept at $0^\circ c$ and $100^\circ c$ respectively, until steady state conditions prevail. If the temperature at B is reduced suddenly to $0^\circ c$ and kept so, while that of A is maintained, find the temperature $u(x,t)$ at distance X from A at time t
4. A rod 30cm, long has its ends A and B kept at $20^\circ c$ and $80^\circ c$ respectively. Until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^\circ c$ and kept so. Find the temperature u(x, t).
5. A rod of length '*l*' has its ends kept at $0^\circ c$ and $200^\circ c$ respectively, until steady state conditions prevail. Thetemperature of the end B is then suddenly reduced to $0^\circ c$ and that of the end A is maintained. Find the subsequent temperature distribution u(x, t) at a distance *x* from the point A and at time *t*.

Non – Zero Boundary Conditions

1. The ends A and B of a rod 1 c.m long have their temperatures kept at 30˚C and 80˚C, until steady state conditions prevail. The temperature of the end B is suddenly reduced to 60˚C and that of A is increased to 40˚C.Find the temperature distribution in the rod after time t.

TWO DIMENSIONAL PLATE

Infinite Plate Problems

1. A rectangular plate with insulated surface is 20 cm wide and so long when compared to its width that it may be considered infinite in length in that introducing an appreciable error. If the temperature of the short edge x=0 is given by $u = \begin{cases} 10y, & 0 \le y \le 10, \\ 10(20-y), & 10 \le y \le 20. \end{cases}$ and the two long edges as well as the other short edge are kept at $0^0 C$, find the steady state temperature distribution in the plate.
2. Aninfinitely long uniform plate is bounded by two parallel edges and an end at right angle to them .the breadth of this edge x = 0 is π, this end is maintained at temperature u=R(πy-y^2) at all points while the other edges are at zero temperature. Find the temperature u(x, y) at any point of the plate in the steady state.

$$u = \begin{cases} 20y, 0 \le y \le 5 \\ 20(10-y)y, 5 \le y \le 10 \end{cases}$$

Find the steady state temperature distribution $u(x, y)$.

3. An infinitely long metal plate in the form of an area is enclosed between the lines y = 0 and y = π for x > 0. The temperature is zero along the edges y = 0 and y = π and at infinity. If the edge x = 0 is kept at a constant temperature T˚C, find the steady state temperature at any point of the plate.

Finite [Square] Plate Problems

1. A square plate of length 20cm has its faces insulated and its edges along x=0,x=20,y=0 and y=20. If the temperature along the upper horizontal edge is given by $u(x,20) = x(20-x)$ when 0 <x< 20, while other edges are maintained at $0^\circ c$. Find the steady state temperature in the plate.
2. A square plate is bounded by the lines $x = 0$, $y = 0$, $x = a$ and$y = a$ and its faces are insulated. The temperature along the edge x = a is maintained at a temperature $100^\circ c$, while the other edges are maintained at $0^\circ c$. Find the steady atate temperature at any point of the plate.

Polar – coordinates

1. A thin semicircular plate of radius 'a' has its bounding diameter kept at temperature zero and its circumference at k. Find the temperature distribution in the steady state

2. A semi circular plate of radius ‘a’ cm has insulated faces and heat flows in plane curve. The bounding diameter is kept at $0^{\circ}c$ and the semi circumference is maintained at temperature 100°C Determine the steady state temperature distribution in the plate.

UNIT IV

Curve Fitting

1. Fit a straight line to the following data by the method of least squares:

x:	5	10	15	20	25
y:	16	19	23	26	30

2. Fit a straight line to the data given below by the method of least square

X:	0	1`	2	3	4
Y:	1	1.8	3.3	4.5	6.3

3. Find a straight line fit of the form $y = a + bx$ by the method of group averages for the following data:

X:	0	5	10	15	20	25
Y:	12	15	17	22	24	30

4. Find by the method of least squares the straight line that test fits the data on the following data.

x :	0	5	10	15	20
y :	7	11	16	20	26

Calculate the value of y when x = 23.

5. Fit a second degree parabola $y = a + bx + cx^2$ to the following data by the method of best squares:

X:	-3	-2	0	3	4
Y:	18		10	2	2	5

6. Fit a parabola of second degree to the following data by the method of least squares.

X:	0	1	2	3	4
Y:	1.0	1.8	1.3	2.5	6.3

7. The job rating efficiency of an employee seems to be relatd to the number of weeks of the employment. For a random sample of 10, employees, the following data were observed.

Job efficiencyX : 55 50 20 55 75 80 90 30 75 70

Weeks of employment: 2 4 1 3 5 9 12 2 7 5

Fit a second degree parabola representing the job efficiency in terms of weeks on employment using method of least squares.

8. Find a relation of the form $y = ab^x$ for the following data using method of least squares:

X:	2	3	4	5	6
Y :	8.31	5.43	3.16	5.212	7.4

LARGE SAMPLES

Single Proportion

1. In a big city, 325 men out of 600 men were found to be smokers. Does ths information support the conclusion that the majority of men in this city are smokers?
2. A random samples of 500 apples was taken from a large consignment and 60 were found to be bad. Obtain the 98% confidence limits for the percentage umber of bad apples in the consignment.
3. In a city, a sample of 1000 people was taken and out of them 540 are vegetarians and the rest are non- vegetarians. Can we say that bpth habits of eating (vegetarian (or) non-vegetarian) are equally popular in the city at

   (i) 1% level of significance

   (ii)5% level of significance

4. Experience has shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50are of top quality. Show that either the production of the day taken was not a representative sample or the hypothesis of 20% was wrong.

5. The guaranteed average life of a certain type of electric light bulbs is 1000 hours with a standard deviation of 125 hours. It is decided to sample the output so a to ensure that 90 percent of the bulbs do not fall short of the guaranteed average by more than 2.4 percent. What must be the minimum size of the sample?

Difference Proportion

1. In a random sample of 1000 persons from the city Coimbatore. 400 are found to be consumers of wheat. In a sample of 800 from the city Madurai, 400 are found to be consumers of wheat. Do these data reveal a significant difference between the two cities, so far as the proportion of wheat consumers is concerned?

2. Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in excise duty, 800 were tea drinkers in a sample of 1200 people. State whether there is a significant decrease in the consumption of tea after the increase in excise duty.
3. 500 articles from a factory are examined and 2% of them are found to be defective. 800 similar articles from a second factory are found to have only 1.5% defective. Can it reasonably be concluded that the product of the first factory are inferior to those of second?

Single Mean

1. The heights of college students in a city are normally distributed with S.D 6 cms. A sample of 100 students has mean height 158 cms. Test the hypothesis that the mean height of college students in the city is 160 cms
2. A sample of 100 people during the past year showed an average life span of 71.8 years. If the standard deviation of the population is 8.9 years, test whether the mean life span today is greater than 70 years.
3. The average marks in Mathematics of a sample of 100 students was 51 with a S.D of 6 marks. Could this have been a random sample from a population with average marks 50?

Difference Mean

1. The average income of persons was Rs. 210 with S.D of Rs. 10 in a sample of 100 people of a city. For another sample of 150 persons the average income was Rs. 220 wih S.D. of Rs. 12. Test whether there is any significant difference between the average income of the localities.
2. A simple sample of heights of 6400 English men has mean of 170 cm and an SD of 6.4 cm, whole a simple sample of heights of 1600 Americans has a mean of 172 cm and an SD of 6.3 cm. Do the data indicate that Americans and on the average, taller than the Englishmen?
3. The mean height of two samples of 1000 and 2000 members are respectively 67.5 and 68 inches. Can they be regarded as drawn from the same population with standard deviation 2.5 inches
4. In a certain factory there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be

120Kgswith a S.D. of 12Kgs while the corresponding figures in a sample of 400 items from the other process are 124 and 14. Obtain the standard error of difference between the two sample means. Is this difference between the average weights of items produced by two machines significant?

Standard Deviation

1. The means of two samples of 1000 and 2000 items are 170cm and 169cm respectively. Can these samples be regarded as drawn from the same population with standard deviation 10 at 5% level of significance?
2. A manufacturer of electric bulbs, according to a certain process, find the standard deviation of the life of lamps to be 100 hours. He wants to change the process, if the new process results in a smaller variation in the life of lamps. In adopting a new process, a sample of 150 bulbs gave a standard deviation of 95 hours. Is the manufacturer justified in changing the process?
3. The standard deviation of a population is 3. Sample of 1000 and 500 are drawn from it and their standard deviations are respectively2.6 and 2 . Is the difference between standard deviations significant?
4. The standard deviation of a random sample of 900 members is 4.6 and that of another independent sample of 1600 members is 4.8. Examine if the two samples could have been drawn from a population with standard deviation 4.0?
5. A sample of heights of 6400 Englishmen have a standard deviation of 6.4 cm while a sample of heights of 1600 Australians have standard deviation 6.3 cm. Is this difference significance?
6. The mean weekly average of a particular item in departmental stores were 146.3 units per store. Ater an advertising campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

UNIT V

T – Test

1. The nine items of sample had the following values: 45, 47, 50, 52, 48, 47, 49, 53 and 51. Does the mean of the nine items differ significantly from one assumed population mean of 47.5?
2. A random sample of 16 values from a normal population showed a mean of 41.5 inches and the sum of squares of deviations from this mean equal to 135 square inches. Show that the assumption of a mean of 43.5 inches for the population is not reasonable.

3. Sandal powder is packed into packets by a machine. A random sample of 12 packets is drawn and their weights are found to be (in kg) 0.49,0.48,0.47,0.48,0.49,0.50,0.51,0.49,0.48,0.50,0.51,0.48. Test if the average packing can be taken 0.5 kg

4. The following table gives the values of protein from Kamgeyam cow's milk and buffalo's milk. By using 't' test examine if these difference are significant

Cow's milk	1.90	1.95	2.00	2.02	1.85	1.85
Buffalo's milk	2.12	2.00	2.20	2.45	2.20	2.10

5. Ten oil tins are taken from an automatic filling machine. The mean weight of the tins is 15.8 kg and standard deviation 0.5 kg. Does the sample mean differ significantly from the intended weight 16 kg.

6. Two independent samples of size 15 are taken from two normal populations. The sample means are 26 and 10 and variations are 7 and 4 respectively. Is the difference between the means significant?

7. Samples of two types of electric bulbs were tested for length of life and the following were obtained

Type	Size	Sample mean	S.D.
I	8	1234hrs	36hrs
II	7	1036hrs	40hrs

Is the difference in the mean sufficient to warrant that type I is superior to type II regarding the length of life?

F – Test

1. Two samples of sizes 9 and 8 gave the sums of squares of deviation from their respective means equal to 160 and 91 respectively . Can they be regarded as drawn from the same normal population?

2. Two independent samples of sizes 6 and 5 from a normal population had the following values of the variables.

   Sample I: 18 13 12 15 12 14

   Sample II: 16 19 13 16 18

   Do the estimate of the population variance differ significantly at 5% level?

3. Two random samples drawn from two normal populations have the values as below:

Sample I :19, 17, 16, 28, 22, 23, 19, 24, 26,

Sample II : 28, 32, 40, 37, 30, ,35, 40, 28, 41, 45, 30, 36. Obtain the estimate of the Variance of the population and test whether the populations have the same variances.

Same Normal Population [Both T and F Test]

1. Two random samples gave the following data:

Sample number	Size	Mean	Variance
1	8	9.6	1.2
2	11	16.5	2.5

Can we conclude that the two samples can be drawn from the sample normal population?

2. Two random samples gave the following results:

Sample	**Size**	**Sample mean**	**Sum of squares of deviations from the mean**
I	10	15	90
II	12	14	108

Test whether the sample could have come from the same normal population.

3. Two random samples gave the following results:

Sample	**Size**	**Sample mean**	**Sum of squares of deviations from the mean**
I	10	15	90
II	12	14	108

Test whether the sample could have come from the same normal population.

Chi – Square Test [Goodness Of Fit]

1. The table below gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week. (5)

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of Accidents	14	18	12	11	15	14

2. A set of 5 identical coins is tossed 320 times and the number if heads appearing each time is recorded :

No of heads	0	1	2	3	4	5
Frequency	14	45	80	112	61	8

Test whether the coins are unbiased at 5% level of significance.

3. For the following distribution, a Poisson fit was made and the expected frequencies are given with the observed frequencies.

Value of x	0	1	2	4	5	6
Frequencies f	142	156	69	27	5	1
Expected frequencies f_x	147	147	74	25	6	1

Test the goodness of fit.

4. Records taken of the number of male and female births in 800 families having four children are as follows:

No. of male births:	0	1	2	3	4
No. of female births:	4	3	2	1	0
No. of families:	32	178	290	236	64

Test whether the data are consistent with the hypothesis that the binomial law holds.

5. In 120 throws of a single die, the following distribution of faces were obtained.

Faces :	1	2	3	4	5	6	Total
Frequency :	30	25	18	10	22	15	120

Compute the statistic you would use to test whether the results constituted reputations of equal probability. We use χ^2 – test.

<u>Chi – Square Test [Attributes]</u>

1. The following data is collected an two characters based on this, can you say that there is no relation between smoking and literacy?

	Smokers	Non Smokers
Literates	83	57
Illiterates	45	68

2. From the following information state whether the condition of the home is associated with the condition of the child.

	Condition of the house	
Condition of the child	**Clean**	**Dirty**
Clean	69	51
Fairly clean	81	20
Dirty	35	44

3. The following data are for a sample of 300 car owners who were classified with respect to age and the number of accidents they had during the past two years. Test whether there is any relationship between these two variables.

	No.of accidents	**1or2 accidents**	**Morethan 2 accidents**
Age less than 30	8	23	14
Age between 30 and 50	21	42	12
Age more than 50	71	90	19

4. Two researchers A and B adopted different techniques while rating the student's level. Can you say that the techniques adopted by them are significant?

Researchers	Below Average	Average	Above Average	Genius	Total
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300

TWO MARK QUESTIONS
UNIT I

2. Formthe p.d.e by eliminating the arbitrary constants 'a' and 'b' from $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$
3. Form the p. d. e. by eliminating the arbitrary function 'f' from $F(z-xy, x^2+y^2)=0$
4. Form partial differential equation by eliminating the arbitrary function from $z=f(x^2-y^2)$ where c is a constant.
5. Solve: $(D^2-2D\ D^1+D'^2)z=0$
6. Find the complete solution of $(p+q)(z-px-qy)=1$
7. From the partial differential equation by eliminating the arbitrary Constant $z=(x+a)(y+b)$
8. Solve: $px+qy=z$
9. From the partial differential equation by eliminating the arbitrary Constant $z=(x^2+a)(y^2+b)$
10. Solve : $(D^2-6DD'+9D'^2)z=0$
11. Find the complete solution of $p^2x^4+y^2zq=2z^2$
12. Form the PDE by eliminating the arbitrary constants from $(x-a)^2+(y-b)^2+z^2=c^2$
13. Solve: $\frac{y^2z}{x}p+xzq=y^2$
14. Solve: $(D^3-3DD^{1^2}+2D^{1^3})z=0$
15. Eliminate the arbitrary constants a and b from $z=ax+by+ab.$
16. Find the particular Integral of $(D^2+4DD')Z=e^x$.

UNIT II

1. State the wave equation in Cartesian form.
2. Write the appropriate solution of the vibration of string equation. How is it chosen?
3. Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement f(x) and initial velocity g(x).
4. What is the constant a^2 in one dimensional wave equation?
5. Write the most suitable solution of the one dimensional wave equation
6. Classify the partial differential equation $u_{xx}+xu_{yy}=0$
7. Explain the method of separation of variables.
8. State any four empirical laws when deriving the wave equation.

9. Write one dimensional wave equation and write all its possible solutions.
10. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is vibrating, each point is given a velocity $3x(l-x)$, at time t=0. Write the mathematical problem.
11. Solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ by the method of separation of variables.
12. Explain the initial value problems.
13. State the assumptions made in the derivatives of one dimensional wave equation.
14. What is boundary value problem

UNIT III

1. When will you use the non-zero boundary conditions in one dimensional heat equation?
2. What is steady state condition prevail in the bar?
3. A rod 30cm long has its ends A and B kept at $20^{\bullet}$C and $70^{\bullet}$C respectively until steady state condition prevails. Find the steady state temperature in the rod.
4. Write any two solutions of the two dimensional heat flow equation under steady state conditions in Cartesian coordinate system.
5. What are the assumptions made while deriving diffusion equation?
6. Write down the equation of heat flow in two dimension in polar co-ordinates.
7. State one dimensional heat flow equation in steady state condition.
8. Write the possible solutions of the 2 dimensional heat equation in Cartesian coordinate
9. Define steady state temperature distribution.
10. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation?
11. Write down the Cartesian form of two dimensional heat flow equations in transient and steady state conditions.
12. Write the polar form of two dimensional heat flow equation in steady state and its various possible solutions
13. Write one dimensional heat flow equation and its various possible solutions.
14. Write the suitable solution of the two dimensional heat flow equation under steady state conditions when the edge x = a has non zero and the other three edges are zero temperature in a square plate.
15. A rod 30cm long has its ends A and B kept at $20^{\bullet}$C and $80^{\bullet}$C respectively until steady state condition prevails. Find the steady state temperature in the rod.

16. Explain the term thermally insulated ends?
17. In steady state conditions derive the solution of one dimensional heat flow equation.

UNIT IV

1. State the principle of least squares?
2. What is the formula for finding the different between sample proportion and population proportion?
3. Define level of significance.
4. A sample of 900 items has the mean 3.4 and S. D. 2.61. Can the sample be regarded as drawn from a population with mean 3.25 at 5% level of significance
5. What is the error committed, when fitting a straight line of the form y = a + bx for the observed values(x_i,y_i, I = 1,2,…n)?
6. Define null hypothesis and alternative hypothesis
7. What is one tailed test ?
8. Define "population
9. Define " Large and Small Samples"
10. Write normal equations for $y = ax^b$
11. Explain type I and type II errors.
12. A coin is tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.
13. Define a residual.

UNIT V

1. Mention any two properties of t – test.
2. Write the probability density function of F-test.
3. Define ψ^2 – test for goodness of fit.
4. What are the assumptions of student's t – test?
5. State the suitable test statistic t for the correlation coefficient of n – paired observed values
6. Define the 't' statistic
7. write down the chisquare statistic for the contingency

a	b
c	d

table

8. Write the uses of F - distribution
9. Write down the any two properties of chi square distribution
10. What are the applications of ψ^2 –test?
11. A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. Find the value of test statistic t, which follows student's t-distribution.

12. How many independent constraints are in a pXq contingency table?
13. A random sample of size 16 has 53 as mean. The sum of the squares of the deviation frommean is 135. Can this be regarde as taken from the population having 56 as mean?
14. Define " Attributes".

ET 41

B.Tech. DEGREE EXAMINATION, APRIL/ MAY 2014

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1.From the partial differential equation by eliminating the arbitrary

Constant $z = (x+a)(y+b)$

2. Solve: $px + qy = z$

3. Write the most suitable solution of the one dimensional wave equation

4.A string is stretched and fastened at two pints x=0 and x=l at a distance 'l' apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time t=0 . Write down the boundary conditions in mathematical form

5.State one dimensional heat flow equation in steady state condition.

6. Write the possible solutions of the 2 dimensional heat equation in Cartesian coordinate

7. Define null hypothesis and alternative hypothesis

8. What is one tailed test ?

9.Define the 't' statistic

10. write down the chisquare statistic for the contingency table

a	b
c	d

SECTION B (5*11 = 55)

Answer FIVE questions, choosing ONE from each unit

UNIT I

11. (a)From the partial differential equation by eliminating the arbitrary unction f and ϕ from $z = f\left(x^2 + y\right) + \varphi\left(x^2 - y)\right)$ (6)

(b) Find the singular integral of $z = px + qx + \left(\frac{q}{p} - p\right)$ (5)

12. (a) Solve: $x(y^2 + z^2)p + y(z^2 + x^2)q = z(y^2 - x^2)$ (5)

(b) $(D^2 - 2DD' + D'^2)\, z = x^2 y^2 e^{x+y}$ (6)

UNIT II

13. Solve by using method of separation of variables the equation $2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$ (11)

(or)

14. A tightly stretched string with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $kx(l - x)$. Find the displacement y(x, t). (11)

UNIT III

15.A thin semicircular plate of radius 'a' has its bounding diameter kept at temperature zero and its circumference at k. Find the temperature distribution in the steady state (11)

(or)

16.A rod of length 1 has its ends A and B kept at $0^\circ c$ and $100^\circ c$ respectively, until steady state conditions prevail. If the temperature at B is reduced suddenly to $0^\circ c$ and kept so, while that of A is maintained, find the temperature $u(x,t)$ at distance X from A at time t (11)

UNIT IV

17. (a) Fit a straight line to the data given below by the method of least square (5)

X:	0	1`	2	3	4
Y:	1	1.8	3.3	4.5	6.3

(b) In a random sample of 1000 persons from the city Coimbatore. 400 are found to be consumers of wheat. In a sample of 800 from the city Madurai, 400 are found to be consumers of wheat. Do these data reveal a significant difference between the two cities, so far as the proportion of wheat consumers is concerned?

(OR)

18. (a) . The heights of college students in a city are normally distributed with S.D 6 cms. A sample of 100 students has mean height 158 cms. Test the hypothesis that the mean height of college students in the city is 160 cms (6)

(b). In a city, a sample of 1000 people was taken and out of them 540 are vegetarians and the rest are non- vegetarians. Can we say that bpth habits of eating (vegetarian (or) non-vegetarian) are equally popular in the city at

(i) 1% level of significance

(ii)5% level of significance (5)

UNIT V

19. (a)Sandal powder is packed into packets by a machine. A random sample of 12 packets is drawn and their weights are found to be (in kg) 0.49,0.48,0.47,0.48,0.49,0.50,0.51,0.49,0.48,0.50,0.51,0.48. Test if the average packing can be taken 0.5 kg (5)

(b) Two researchers A and B adopted different techniques while rating the student's level. Can you say that the techniques adopted by them are significant?

Researchers	Below Average	Average	Above Average	Genius	Total
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300

(OR)

20. (a) The table below gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week. (5)

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of Accidents	14	18	12	11	15	14

(b) The following table gives the values of protein from Kamgeyam cow's milk and buffalo's milk. By using 't' test examine if these difference are significant

Cow's milk	1.90	1.95	2.00	2.02	1.85	1.85
Buffalo's milk	2.12	2.00	2.20	2.45	2.20	2.10

ET 41

B.Tech. DEGREE EXAMINATION, APRIL/MAY 2011

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

7. Form partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$ where c is a constant.
8. Solve: $(D^2 - 2D\ D^1 + D'^2)z = 0$
9. Write the appropriate solution of the vibration of string equation. How is it chosen?
10. Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement f(x) and initial velocity g(x).
11. A rod 30cm long has its ends A and B kept at $20^\bullet$ C and $70^\bullet$ C respectively until steady state condition prevails. Find the steady state temperature in the rod.
12. Write any two solutions of the two dimensional heat flow equation under steady state conditions in Cartesian coordinate system.
13. Define level of significance.
14. A sample of 900 items has the mean 3.4 and S. D. 2.61. Can the sample be regarded as drawn from a population with mean 3.25 at 5% level of significance
15. Define ψ^2 – test for goodness of fit.
16. What are the assumptions of student's t-test?

Section B-(5X11=55marks)

Answer five questions choosing one from each unit

Unit I

17. a) Form partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2+(y-b)^2+z^2=c^2$

b) Solve: $(2D^2-5DD^1+2D^{1^2})z=5\sin(2x+y)$

Or

18. a) Solve: $x(y^2+z)p+y(x^2+z)q=z(x^2-y^2)$

b) Solve: $p(1+q)=qz$

Unit II

19. If a string of length l is initially at rest in its equilibrium position and each of its points is given a velocity v such that $v=\begin{cases} cx & 0<x<l/2 \\ c(l-x) & l/2<x<l \end{cases}$ Find the displacement of the string at any time t.

Or

20. A string is tightly stretched and its ends are fastened at two points x = 0 and x = 2l. The midpoint of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string.

Unit III

21. Find the solution of the equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ that satisfies the condition.

$u(0,t)=0, u(l,t)=0\, fort\geq 0 and\ u(x,0)=u_0\ for\, all\, x$

Or

22. An infinitely long uniform plate is bounded by two parallel edges and an end at right angle to them . the breadth of this edge x = 0 is π, this end is maintained at temperature u=R(πy-y^2) at all points while the other edges are at zero temperature. Find the temperature u(x, y) at any point of the plate in the steady state.

$$u=\begin{cases} 20y, 0\leq y\leq 5 \\ 20(10-y)y, 5\leq y\leq 10 \end{cases}$$

Find the steady state temperature distribution $u(x,y)$.

Unit IV

23. Fit a second degree parabola $y = a + bx + cx^2$ to the following data by the method of best squares:

X:	-3	-2	0	3	4
Y:	18		10	2	2	5

Or

24. a) In a big city, 325 men out of 600 men were found to be smokers. Does ths information support the conclusion that the majority of men in this city are smokers?

b) The average income of persons was Rs. 210 with S.D of Rs. 10 in a sample of 100 people of a city. For another sample of 150 persons the average income was Rs. 220 wih S.D. of Rs. 12. Test whether there is any significant difference between the average income of the localities.

Unit V

25. Two random samples gave the following results:

Sample	**Size**	**Sample mean**	**Sum of squares of deviations from the mean**
I	10	15	90
II	12	14	108

Test whether the sample could have come from the same normal population.

Or

26. From the following information state whether the condition of the home is associated with the condition of the child.

	Condition of the house	
Condition of the child	**Clean**	**Dirty**
Clean	69	51
Fairly clean	81	20
Dirty	35	44

ET 41

B.Tech. DEGREE EXAMINATION, APRIL/MAY 2012

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Form partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$
2. Find the complete solution of $(p+q)(z - px - qy) = 1$
3. What is the constant a^2 in one dimensional wave equation?
4. Whatare the possible solutions of the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$?
5. What are the assumptions made while deriving diffusion equation?
6. Write down the equation of heat flow in two dimension in polar co-ordinates.
7. What is the error committed, when fitting a straight line of the form y = a + bx for the observed values(x_i,y_i, I = 1,2,…n)?
8. Define null hypothesis.
9. State the suitable test statistic t for the correlation coefficient of n-paired observed values
10. Define ψ^2 – test for goodness of fit.

Section B-(5X11=55marks)

Answer five questions choosing one from each unit

Unit I

11. a) Find the general solution of $(3z - 4y)p + (4x - 2z)q = (2y - 3x)$

    b) Solve: $(D^3 - 7DD^{1^2} - 6D^{1^3})z = e^{2x+y} + \sin(x + 2y)$

Or

12. a) Solve: $z^2(p^2x^2 + q^2) = 1$

    b) Solve: $(D+1)(D + D^1 - 1)z = 5e^x$

Unit II

13. a)Solve by method of separation of variables $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ when $u(x,0) = 4e^{-x}$

b)The points of trisection of a string of length *'2l'* are pulled aside through a distance *'d'* on opposite sides of the position of equilibrium and the string is released from rest. Write the mathematical form of this physical situation

Or

14. a)A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is vibrating, each point is given a velocity $\lambda x(l-x)$, find the displacement of the string at any point and at any time.

Unit III

15. A rod 30cm, long has its ends A and B kept at $40^{\circ}c$ and $100^{\circ}c$ respectively. Until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^{\circ}c$ and kept so. Find the temperature u(x, t).

Or

16. A square plate of length 20cm has its faces insulated and its edges along x=0,x=20,y=0 and y=20. If the temperature along the upper horizontal edge is given by $u(x,20)=x(20-x)$ when 0 <x< 20, while other edges are maintained at $0^{\circ}c$. Find the steady state temperature in the plate.

Unit IV

17. a) A random samples of 500 apples was taken from a large consignment and 60 were found to be bad. Obtain the 98% confidence limits for the percentage umber of bad apples in the consignment.

b)Fit a parabola of second degree to the following data by the method of least squares.

X:	0	1	2	3	4
Y:	1.0	1.8	1.3	2.5	6.3

Or

18. a)The means of two samples of 1000 and 2000 items are 170cm and 169cm respectively. Can these samples be regarded as drawn from the same population with standard deviation 10 at 5% level of significance?

b) A manufacturer of electric bulbs, according to a certain process, find the standard deviation of the life of lamps to be 100 hours. He wants to change the process, if the new process results in a smaller variation in the life of lamps. In adopting a new process, a

sample of 150 bulbs gave a standard deviation of 95 hours. Is the manufacturer justified in changing the process?

Unit V

19. a) A random sample of 16 values from a normal population showed a mean of 41.5 inches and the sum of squares of deviations from this mean equal to 135 square inches. Show that the assumption of a mean of 43.5 inches for the population is not reasonable.

b)Two independent samples of sizes 6 and 5 from a normal population had the following values of the variables.

Sample I: 18 13 12 15 12 14

Sample II: 16 19 13 16 18

Do the estimate of the population variance differ significantly at 5% level?

Or

20. a) A semi circular plate of radius 'a' cm has insulated faces and heat flows in plane curve. The bounding diameter is kept at $0^{\circ}c$ and the semi circumference is maintained at temperature 100° C Determine the steady state temperature distribution in the plate.

b) The following data are for a sample of 300 car owners who were classified with respect to age and the number of accidents they had during the past two years. Test whether there is any relationship between these two variables.

	No.of accidents	**1or2 accidents**	**Morethan 2 accidents**
Age less than 30	8	23	14
Age between 30 and 50	21	42	12
Age more than 50	71	90	19

4604054

B.Tech. DEGREE EXAMINATION, NOVEMBER 2013.

Fourth Semester

Mechanical Engineers

MATHEMATICS – IV

Time: Three hours Maximum : 75 marks

SECTION A – (10 X 2 = 20 marks)

Answer ALL questions.

2. Eliminate the arbitrary constants a and b from $z = ax + by + ab.$
3. Find the particular Integral of $(D^2 + 4DD')Z = e^x$.
4. Explain the initial value problems.
5. State the assumptions made in the derivatives of one dimensional wave equation.
6. Explain the term thermally insulated ends?
7. In steady state conditions derive the solution of one dimensional heat flow equation.
8. Define a residual.
9. Define "large sample".
10. Write the uses of F – distribution.
11. Define " Attributes".

SECTION B –(5 x 11 =55 marks)

Answer all questions, choosing one question from each unit.

UNIT I

12. Solve $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \frac{x-y}{xy}$.

(Or)

13. Solve $\left(D^3 - 4D^2D' + 4DD'^{2'}\right)z = 6\sin(3x + 6y).$

UNIT II

14. Solve by using method of separation of variables the equation $x^2q + y^3p = 0$.

(Or)

15. A string is stretched and fastended to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k\left(lx - x^2\right)$ from which it is released at time $t = 0$ .Find the displacement of any point on the string at a distance of x from one end at time t.

UNIT III

16. The ends A and B of a rod 1 c.m long have their temperatures kept at 30˚C and 80˚C, until steady state conditions prevail. The temperature of the end B is suddenly reduced to 60˚C and that of A is increased to 40˚C.Find the temperature distribution in the rod after time t.

(Or)

17. An infinitely long metal plate in the form of an area is enclosed between the lines y = 0 and y = π for x > 0. The temperature is zero along the edges y = 0 and y = π and at infinity. If the edge x = 0 is kept at a constant temperature T˚C, find the steady state temperature at any point of the plate.

UNIT IV

18. Find by the method of least squares the straight line that test fits the data on the following data.

x :	0	5	10	15	20
y :	7	11	16	20	26

Calculate the value of y when x = 23.

(Or)

19. A sample of heights of 6400 Englishmen have a standard deviation of 6.4 cm while a sample of heights of 1600 Australians have standard deviation 6.3 cm. Is this difference significance?

UNIT V

20. Ten oil tins are taken from an automatic filling machine. The mean weight of the tins is 15.8 kg and standard deviation 0.50 kg. Does the sample mean differ significantly from the intended weight 16 kg.

(Or)

21. In 120 throws of a single die, the following distribution of faces were obtained.

Faces :	1	2	3	4	5	6	Total
Frequency :	30	25	18	10	22	15	120

Compute the statistic you would use to test whether the results constituted reputations of equal probability. We use χ^2 – test.

ET 41

B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2010

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Solve: $yq + xp = z$
2. Find the complete solution of $p^2x^4 + y^2zq = 2z^2$
3. State any four empirical laws when deriving the wave equation.
4. Write one dimensional wave equation and write all its possible solutions.

5. Write down the Cartesian form of two dimensional heat flow equations in transient and steady state conditions.
6. Write the polar form of two dimensional heat flow equation in steady state and its various possible solutions
7. Define null and alternate hypothesis
8. A sample of 900 members from a normal population with S. D. 2.61cms has a mean 3.5cms. Find the 95% fiducial limits for the population mean.
9. Give any two properties of the t-distribution.
10. What are the applications of ψ^2 –test?

Section B-(5X11=55marks)

Answer five questions choosing one from each unit

Unit I

11. a) Form partial differential equation by eliminating the arbitrary function from $f(xy+z^2, x+y+z)=0$

    b) Solve: $(D^2 - D^{1^2})z = e^{x-y} + x^2 y$

Or

12. a) Solve: $9(p^2 z + q^2) = 4$

    b) Solve: $(D^2 + 2DD' + D'^2 - 2D - 2D^1)z = e^{x-y} + x^2 y$

Unit II

13. a)Solve by method of separation of variables $3\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ when $u(x,0) = 4e^{-x}$

    b) The points of trisection of a string of length *'2l'* are pulled aside through a distance *'d'* on opposite sides of the position of equilibrium and the string is released from rest. Write the mathematical form of this physical situation

Or

14. A taught string of length 20cms fastened at both ends is displaced from its position of equilibrium, by imparting to each of its points an initial velocity given by

$$v = \begin{cases} x & 0 < x < 10 \\ 20 - x, & 10 < x < 20 \end{cases}$$

x being the distance from one end. Determine the displacement of the string at any subsequent time.

Unit III

15. An insulated rod of length 'l' has its ends A and B kept at $0^{\circ}c$ and $100^{\circ}c$ respectively, until steady state conditions prevail. The temperature of the end B is then suddenly reduced to $0^{\circ}c$ and that of the end A is maintained. Find the subsequent temperature distribution u(x, t) at a distance x at time t.

Or

16. A square plate is bounded by the lines $x = 0$, $y = 0$, $x = a$ and $y = a$ and its faces are insulated. The temperature along the edge x = a is maintained at a temperature $100^{\circ}c$, while the other edges are maintained at $0^{\circ}c$. Find the steady atate temperature at any point of the plate.

Unit IV

17. a) The following table shows the recorded data of test scores made by salesman on an intelligence test and their daily sales.

Salesman	1	2	3	4	5	6	7	8	9	10
Test scores	40	70	50	60	80	50	90	40	60	60
Sales in 1000 rupees	2.5	6.0	4.5	5.0	4.5	2.0	5.5	3.0	4.5	3.0

Determine the regression line of sales on test scores and estimate the most probable daily sales volume if a salesman makes a score of 70.

b) In a certain factory there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120Kgswith a S.D. of 12Kgs while the corresponding figures in a sample of 400 items from the other process are 124 and 14. Obtain the standard error of difference between the two sample means. Is this difference between the average weights of items produced by two machines significant?

Or

18. a) Experience has shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50are of top quality. Show that either the production of the day taken was not a representative sample or the hypothesis of 20% was wrong.

b) The standard deviation of a random sample of 900 members is 4.6 and that of another independent sample of 1600 members is 4.8. Examine if the two samples could have been drawn from a population with standard deviation 4.0?

Unit V

19. a)The nine items of sample had the following values: 45, 47, 50, 52, 48, 47, 49, 53 and 51. Does the mean of the nine items differ significantly from one assumed population mean of 47.5?

b)Two samples of sizes 9 and 8 gave the sums of squares of deviation from their respective means equal to 160 and 91 respectively . Can they be regarded as drawn from the same normal population?

Or

20. a) The ranking of ten students in two subjects A and B are as follows.

A	3	5	8	4	7	10	2	1	6	9
B	6	4	9	8	1	2	3	10	5	7

Find the Spearman's rank correlation between the two ranks and test the validity of the

b) For the following distribution, a Poisson fit was made and the expected frequencies are given with the observed frequencies.

Value of x	0	1	2	4	5	6
Frequencies f	142	156	69	27	5	1
Expected frequencies f_x	147	147	74	25	6	1

Test the goodness of fit.

ET 41

B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2011

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Form the PDE by eliminating the arbitrary constants from $(x-a)^2+(y-b)^2+z^2=c^2$
2. Solve: $\frac{y^2z}{x}p+xzq=y^2$
3. State any four empirical laws when deriving the wave equation.
4. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is vibrating, each point is given a velocity $3x(l-x)$, at time t=0. Write the mathematical problem.
5. Write one dimensional heat flow equation and its various possible solutions.
6. Write the suitable solution of the two dimensional heat flow equation under steady state conditions when the edge x = a has non zero and the other three edges are zero temperature in a square plate.
7. Write normal equations for $y = ax^b$
8. Explain type I and type II errors.
9. A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. Find the value of test statistic t, which follows student's t-distribution.
10. How many independent constraints are in a pXq contingency table?

Section B-(5X11=55marks)

Answer five questions choosing one from each unit

Unit I

11. a) Form partial differential equation by eliminating the arbitrary function from $\phi(x+y+z, x^2+y^2-z^2)=0$

    b) Solve: $(D^3-2D\ D^1)z=3x^2y+\sin(x+2y)$

Or

12. a) Solve: $z^2(p^2+q^2)=(x^2+y^2)$

    b) Solve: $(D^2-2DD'+D'^2-3D+3D^1+2)z=e^{2x-y}$

Unit II

13. a)Solve by method of separation of variables $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=3u$ where $u(0,t) = e^{-5t}$

    b) Find all the possible solutions of the PDE $y_{tt}=a^2y_{xx}$ using separation of variables method.

Or

14. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $f(x)=\begin{cases} x, & 0<x<l/2 \\ l-x & l/2<x<l \end{cases}$. It is released from rest from this position, find the displacement of the string at any point x from one end and at any time t.

Unit III

15. A rod 30cm, long has its ends A and B kept at $20^{\circ}c$ and $80^{\circ}c$ respectively. Until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^{\circ}c$ and kept so. Find the temperature u(x, t).

Or

16. An infinitely long rectangular plate with insulated surface is 10 cm wide. The two edges and one short edge are kept at zero temperature, while the other short edge $x=0$ is kept at temperature given by

$$u=\begin{cases} 20y, 0\le y\le 5 \\ 20(10-y)y, 5\le y\le 10 \end{cases}$$

Find the steady state temperature distribution $u(x,y)$.

Unit IV

17. The job rating efficiency of an employee seems to be relatd to the number of weeks of the employment. For a random sample of 10, employees, the following data were observed.

Job efficiencyX	:	55	50	20	55	75	80	90	30	75	70
Weeks of employment:		2	4	1	3	5	9	12	2	7	5

Fit a second degree parabola representing the job efficiency in terms of weeks on employment using method of least squares.

Or

18. a) Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in excise duty, 800 were tea drinkers in a sample of 1200 people. State whether there is a significant decrease in the consumption of tea after the increase in excise duty.

b) The guaranteed average life of a certain type of electric light bulbs is 1000 hours with a standard deviation of 125 hours. It is decided to sample the output so a to ensure that 90 percent of the bulbs do not fall short of the guaranteed average by more than 2.4 percent. What must be the minimum size of the sample?

Unit V

19. a) The mean weekly average of a particular item in departmental stores were 146.3 units per store. Ater an advertising campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

b) Two random samples gave the following results:

Sample	**Size**	**Sample mean**	**Sum of squares of deviations from the mean**
I	10	15	90
II	12	14	108

Test whether the sample could have come from the same normal population.

Or

20. a)Samples of two types of electric bulbs were tested for length of life and the following were obtained

Type	**Size**	**Sample mean**	**S.D.**
I	8	1234hrs	36hrs
II	7	1036hrs	40hrs

Is the difference in the mean sufficient to warrant that type I is superior to type II regarding the length of life?

b) The following table gives the number of aircraft accidents that occurred during the seven days of the week. Find whether the accidents are uniformly distributed over the week.

Days of the week	: Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of accidents	:14	17	12	11	15	14	15

ET 41

B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Form partial differential equation by eliminating the arbitrary constants from $(x-a)^2+(y-b)^2+z^2=1$
2. Solve: $(D^3-3DD^{1^2}+2D^{1^3})z=0$
3. Write the possible solutions of the the transverse vibrations of a string.
4. Solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ by the method of separation of variables.
5. A rod 30cm long has its ends A and B kept at $20^{\bullet}$ C and $80^{\bullet}$ C respectively until steady state condition prevails. Find the steady state temperature in the rod.
6. Write any two solutions of the two dimensional heat flow equation under steady state conditions in Cartesian coordinate system..
7. A coin is tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.
8. Write down the principle of least squares.
9. what are the applications of ψ^2 –test?
10. A random sample of size 16 has 53 as mean. The sum of the squares of the deviation frommean is 135. Can this be regarde as taken from the population having 56 as mean?

Section B-(5X11=55marks)

Answer five questions choosing one from each unit

Unit I

11. a) Find the complete and singular solution of $z=px+qy+p^2-q^2$

    b) Solve: $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$

Or

12. a) Solve: $(D^2+4DD^1-5D^{1^2})z=3e^{2x-y}+\sin(x-2y)$

    b) Solve:Form the PDE by eliminating arbitrary function from $z=f(2x+3y)+g(2x+y)$

Unit II

13. A string is tightly stretched and its ends are fastened at two points x = 0 and x = l. The midpoint of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string.

Or

14. A taught string of length 20cms fastened at both ends is displaced from its position of equilibrium, by imparting to each of its points an initial velocity given by

$$v=\begin{cases} x & 0<x<10 \\ 20-x, & 10<x<20 \end{cases}$$

x being the distance from one end. Determine the displacement of the string at any

subsequent time.

Unit III

15. A rod of length 'l' has its ends kept at $0^\circ c$ and $200^\circ c$ respectively, until steady state conditions prevail. Thetemperature of the end B is then suddenly reduced to $0^\circ c$ and that of the end A is maintained. Find the subsequent temperature distribution u(x, t) at a distance x from the point A and at time t.

Or

16. A long rectangular plate has its surfaces insulated and the 2 long sides as well as one of the short edges are maintained at $0^\circ c$. Find an expression for the steady state temperature u(x,y), the short edge y=0 is kept at $u_0{}^\circ c$.

Unit IV

17. Find a relation of the form $y=ab^x$ for the following data using method of least squares:

X:	2	3	4	5	6
Y :	8.315.433.1	65.2127.4			

Or

18. a)500 articles from a factory are examined and 2% of them are found to be defective. 800 similar articles from a second factory are found to have only 1.5% defective. Can it reasonably be concluded that the product of the first factory are inferior to those of second?

    b)The average marks in Mathematics of a sample of 100 students was 51 with a S.D of 6 marks. Could this have been a random sample from a population with average marks 50?

Unit V

19. Two random samples drawn from two normal populations have the values as below:

    Sample I : 19, 17, 16, 28, 22, 23, 19, 24, 26,

    Sample II : 28, 32, 40, 37, 30, ,35, 40, 28, 41, 45, 30, 36. Obtain the estimate of the variance of the population and test whether the populations have the same variances.

Or

20. Records taken of the number of male and female births in 800 families having four children are as follows:

No. of male births:	0	1	2	3	4
No. of female births:	4	3	2	1	0
No. of families:	32	178	290	236	64

Test whether the data are consistent with the hypothesis that the binomial law holds.

ET 41

B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. From the partial differential equation by eliminating the arbitrary

Constant $z=(x^2+a)(y^2+b)$

2. Solve : $(D^2-6DD'+9D'^2)z=0$

3. Classify the partial differential equation $u_{xx}+xu_{yy}=0$

4. Explain the method of separation of variables.

5. Define steady state temperature distribution.

6. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation?

7. Define "population

8. Define " Large and Small Samples"

9. Write the uses of F- distribution

10. Write down the any two properties of chi square distribution

Part B (5*11=55)

Answer ONE full question from each unit

UNIT I

11. (a) Solve: $p^2+pq=z^2$ (6)

(b)Solve: $(D^2+3DD'-4D'^2)Z=\sin y$ (5)

OR

12.(a) Solve: $(3z-4y)\frac{\partial z}{\partial x}+(4x-2z)\frac{\partial z}{\partial y}=(2y-3x)$ (6)

(b) Solve: $(D^2-2DD')z=e^{2x}+x^{3y}$ (5)

UNIT II

13. (a) . Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, u(0, y) = 8e^{-3y}$ by using method of separation of variables (6)

(b)Classify the following PDE:

(i) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ (one dimensional heat equation)

(ii) $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ (one dimensional wave equation)

(iii) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial y^2} = 0$ (one dimensional heat equation) (5)

(or)

14. A string is tightly stretched and its ends are fastened at two points x=0 and x=l . The midpoint of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string at any during the subsequent motion (11)

UNIT III

15. Solve the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, subject to the boundary conditions

$u(0,t) = 0, u(l,t) = 0, u(x,0) = x$ (11)

(Or)

16.A rod of length 1 has its ends A and B kept at $0^\circ c$ and $100^\circ c$ respectively, until steady state conditions prevail. If the temperature at B is reduced suddenly to $0^\circ c$ and kept so, while that of A is maintained, find the temperature $u(x,t)$ at distance X from A at time t (11)

UNIT IV

17. (a) Find a straight line fit of the form $y = a + bx$ by the method of group averages for the following data:

X:	0	5	10	15	20	25
Y:	12	15	17	22	24	30

(b) A sample of 100 people during the past year showed an average life span of 71.8 years. If the standard deviation of the population is 8.9 years, test whether the mean life span today is greater than 70 years.

18. (a) The mean height of two samples of 1000 and 2000 members are respectively 67.5 and 68 inches. Can they be regarded as drawn from the same population with standard deviation 2.5 inches .(6)

(b) The standard deviation of a population is 3. Sample of 1000 and 500 are drawn from it and their standard deviations are respectively2.6 and 2 . Is the difference between standard deviations significant? (5)

UNIT V

19. (a)Ten oil tins are taken from an automatic filling machine. The mean weight of the tins is 15.8 kg and standard deviation 0.5 kg. Does the sample mean differ significantly from the intended weight 16 kg. (6)

(b) Two independent samples of size 15 are taken from two normal populations. The sample means are 26 and 10 and variations are 7 and 4 respectively. Is the difference between the means significant? (5)

(or)

20.A set of 5 identical coins is tossed 320 times and the number if heads appearing each time is recorded :

No of heads	0	1	2	3	4	5
Frequency	14	45	80	112	61	8

Test whether the coins are unbiased at 5% level of significance. (11)

ET 41

B.Tech. DEGREE EXAMINATION, APRIL\MAY 2015

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Form the partial differential equation by eliminating a and b from $Z = (x^2+ a^2)(y^2+ b^2)$.
2. Solve $(D^3-3DD'^2+2D'^3)Z = 0$.
3. What are the possible solutions of one dimensional wave equation?
4. In the wave equation $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$ what does α^2 stand for?

5. A rod 30 cm along has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. Find the steady state temperature in the rod.
6. Write any two solutions of the Laplace equations obtained by the method of separation of variables.
7. State the principle of least squares.
8. Define null hypothesis and alternative hypothesis.
9. What is meant by standard error?
10. Give any two uses of *t*-distribution.

Part B (5*11=55)

Answer ONE full question from each unit

UNIT I

11. (a)Solve $x(y-z)p + y(z-x)q = z(x-y)$. (6)
    (b) Solve $(D^2 - 2DD' + D'^2)z = cos(x-3y)$. (5)

Or

12. (a) Find the complete integral of $x^2p^2+y^2q^2 = z^2$.
    (b) Solve $\left(D^3 - 7DD'^2 + D'^3\right)z = x^2y + \sin(x+2y)$.

UNIT-II

13. A lightly stretched flexible string has its ends fixed at x=0 and x=l. At time t=0, the string is given a shape defined by $f(x) = kx^2(l-x)$, where k is a constant, and then released from rest. Find the displacement of any point x of the string at any time t>0.

Or

14. A lightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, then show that $y(x,t) = \frac{8\lambda l^3}{\pi^4\alpha}\sum_{n=1,3,5}^{\infty}\frac{1}{n^4}sin\frac{n\pi x}{l}sin\frac{n\pi at}{l}$.

UNIT-III

15. Find the solution to the equation $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions
    (a)$u(0,t) = 0$
    (b) $u(l,t) = 0, t > 0$
    (c) $u(x,0) = \begin{cases} x, & 0 < x < l/2 \\ l-x, & l/2 < x < l \end{cases}$

Or

16. Find the steady state temperature at any point of a square plate whose three edges are kept at 0°C and the other one edges are kept at the constant temperature 100°C.

UNIT-IV

17. Fit a parabola $y - ax^2 + bx + c$ by the method of least squares to the following data

x:	10	12	15	23	20
y:	14	17	23	25	21

Or

18. Before an increase excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant decrease in the consumption of tea after the increase in duty.

UNIT-V

19. Two independent samples of eight and seven item respectively had the following values of the variables.

Sample I:	9	11	13	11	15	9	12	14
Sample II:	10	12	10	14	9	8	10	

Do the two estimates of population variance differ significantly at 5% level of significance?

Or

20. Fit a Poison distribution to the following data and also test the goodness of fit

x:	0	1	2	3	4	5	Total
y:	142	156	69	27	5	1	400

ET 41

B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER2015

Fourth Semester

MATHEMATICS IV

Time: Three hours **Maximum: 100 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Form the PDE by eliminating the arbitrary constants from $Z = (x - a)2 + (y - b)2 + 1$.
2. Find the particular integral of $(D2 - 3DD' + 2D'2)Z = sin(x + 5y)$.
3. State the assumptions made in the derivation of one dimensional wave equation.
4. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation?
5. State the Fourier law of heat conduction.
6. How many boundary conditions are required to solve completely $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$?
7. Write down the normal equations to fit a parabola $y = ax^2 + bx + c$ by the method of least squares.
8. Define Type I and Type II errors.
9. Give the 95% confidence interval of the population mean in terms of the mean and S.D. of a small sample.
10. What is the use of F- distribution.

Part B (5*11=55)

Answer ONE full question from each unit

UNIT I

11. (a)Find the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2 + z^2)$. (5)

(b) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$. (6)

Or

12. (a) Find the complete integral of $(x^2+y^2 + yz)p + (x^2+y^2 - xz)q = z(x + y)$.

(b) Solve $\left(D^2 - 2DD' + D'^2\right)z = \cos(\mathrm{x} - 3\mathrm{y})$.

UNIT-II

13. A lightly stretched flexible string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0)$=$y_0 sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.

Or

14. A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocities v where

$$v=\begin{cases} \frac{Cx}{l}, 0 < x < l \\ \frac{C}{l}(2l - x), l < x < 2l \end{cases}$$

x being the distance from an end point. Find the displacement of the string at any time.

UNIT-III

15. A rod of length l has its ends A and B kept at 0°C and 100°C until steady state condition prevail. If the temperature at B is reduced suddenly to 0°C and kept so while that of A is maintained. Find the temperature $u(x, t)$at a distance x from A and at time t

Or

16. A long rectangular plate has its surfaces insulated and the 2 long sides as well as one of the short edge are maintained at 0°C. Find an expression for the steady state temperature $u(x, y)$, the short edge $y = 0$ is kept at u_0°C

UNIT-IV

17. Find the curve of best fit of the type $y = ae^{bx}$ to the following data by least squares method.

x:	1	5	7	9	12
y:	10	15	12	15	21

Or

18. The average marks scored by 32 boys is 72 with a S.D. of 8, while that for 36 girls is 70 with a S.D. of 6. Test at 1% level of significance whether the boys perform better than girls.

UNIT-V

19. Two samples of sizes nine and eight gave the sums of squares of deviations from their respective means equal to 160 and 91 respectively. Can they be regarded a s drawn from the same normal population?

Or

20. The following table gives the number of air-craft accidents occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Day:	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents:	15	19	13	12	16	15

B.Tech. DEGREE EXAMINATION, MAY 2016

Fourth Semester

MATHEMATICS IV

(2013-2014 Batch onwards)

Time: Three hours **Maximum: 75 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. From the partial differential equation from $z = f(2x-6y)$

2. Find the complete integral of $z = px + qy + p^2 - q^2$.

3. Write all possible solutions of one dimensional wave equation.

4. What are the assumptions made in deriving the one dimensional wave equation?

5. A rod of 30 cm long its ends A and B kept at $20^\circ c$ and $80^\circ c$ until steady state conditions prevails.Define steady state temperature distribution.

6. Give the three possible solutions of two dimensional steady state heat flow equation.

7. Define null hypothesis and alternate hypothesis.

8. Mention the various steps involved in testing of hypothesis.

9. Write any two applications of t-test.

10. Define ψ^2 – test for goodness of fit.

Part B (5x11=55)

Answer ONE full question from each unit

UNIT I

11. Solve the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

(or)

12. Solve $(D^2 + 4DD' - 5D'^2)z = 3e^{2x-y} + \sin(x-2y)$.

UNIT II

13. A string is stretched and fastened to two points *l* apart. Motion is started by displacing the string into the form $k(lx - x^2)$ from which it is released at time t=0. Find the displacement of any point on the string at a distance from one end at time 't'.

(or)

14. A string is stretched between two fixed points at a distance apart and the points on the string are given initial velocity V, where $v = \begin{cases} \frac{c}{l}x, & 0 < x < l \\ \frac{c}{l}(2l - x), & l < x < 2l \end{cases}$. Find the displacement .

UNIT III

15 A metallic bar10 cm long with insulated sides has its ends A and B kept at $20^\circ c$ and $40^\circ c$ respectively, until steady state conditions prevail. The temperature atA is then suddenly raised to $50^\circ c$ and B is lowered to $10^\circ c$. Find the subsequent temperature distribution.

(or)

16. A rectangular plate with insulated surface is10 cm wide and so long when compared to its width that it may be considered infinite in length. The temperature along the short edge y=0 is given by $u(x,0) = \begin{cases} 20x, & 0 < x < 5, \\ 20(10 - x), & 5 < y < 10. \end{cases}$ The other short edge are kept at $0^0 C$, find the steady state temperature distribution in the plate.

UNIT IV

17. Fit a parabola $y = ax^2 + bx + c$ in least squares sense to the data:

X:	10	12	15	23	20
Y:	14	17	23	25	21

18. A machine produced 20 defective units in a sample of 400. After overhauling the machine it produced 10 defective units in a batch of 300. Has the machine improved in production due to Overhauling? Test it a 5% L.O.S.

UNIT V

19. A random sample of 10 boys had the following I.Q's70,120,110,101,88,83,95,107,100. Do these data support the assumption of population mean I.Q of 100?.

(or)

20 . The following table gives the number of aircraft accidents that occur during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	18	12	11	15	14	84

B.Tech. DEGREE EXAMINATION, NOVEMBER 2016

Fourth Semester

MATHEMATICS IV

(2009-2013 Batch onwards)

Time: Three hours **Maximum: 75 marks**

PART A(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1. Solve $(D^2+5DD+4D'^2)z=0.$

2. Form the PDE by eliminating the arbitrary functions from $z=f(x^2+y^2)$.

3. State one dimensional wave equation.

4. Write the boundary conditions for solving the string equation, if the string is subjected to initial displacement f(x) and initial displacement g(x).

5. Define steady state temperature distribution.

6. Mention the steady state solution of the one –dimensional heat equation.

7. Define two tailed test.

8. What are Type I and Type II errors?

9. Give any two uses of t-test.

10. state any two applications of chi-square test.

Part B (5x11=55)

Answer ONE full question from each unit.

UNIT I

11. (a) Find the partial differential equation of all spheres of the same radius 'c' having their centres on the yz plane. (6)

(b) Find the complete solution of the partial differential equation $p^2+q^2-4pq=0$ (5)

(or)

12. (a) Solve $(mz-ny)p+(nx-lz)q=ly-mx.$

(b) Solve $(D^2+4DD'-5D'^2)z=xy+\sin(x-2y).$

UNIT II

13. A tightly stretched string is fixed at two points $x=0$ and $x=l$, motion is started by displacing the string into the form $y = k(lx - x^2)$. From which it is released at time $t=0$. Find the displacement of an string at a distance x from one end at any time t.

(or)

14. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

UNIT III

15 An infinitely long metal plate in the form of an area is enclosed between the lines $y=0$ and $y=\pi$ for positive values x. The temperature is zero along the edges $y=0$, $y=\pi$ and the edge at infinity. If the edge $x=0$ is kept at temperature *ky*, find the steady state temperature distribution in the plate.

(or)

16. The boundary diameter of a semi circular plate of radius 'a' cm is kept at $0^0 C$ and the temperature along the semicircular boundary is given by $u(a,\theta) = \begin{cases} 50\theta, & 0<\theta<\frac{\pi}{2}, \\ 50(\pi-\theta), & \frac{\pi}{2}<\theta<\pi. \end{cases}$. Fnd the steady state temperature function $u(r,\theta)$.

UNIT IV

17. (a)The means of two large samples of size 2000 and 1000 are 68.0 gm and 67.5 gm respectively. Can the sample be regarded as drawn from the same population with standard deviation 12.25 gm?

(b) Fit a curve of the form $y = ab^x$ for the following data by the method of least squares:

x:	1	2	3	4	5	6
y :	151	100	61	50	20	8

18. (a) A manufacture claims that only 4% of his products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim at 5% level of significance.

(b) The standard deviation of a random sample of 1000 is found to be 2.6 and the standard deviation of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same standard deviation.

UNIT V

19. (a) On the basis of information noted below, find out whether the new treatment is comparatively superior to the conventional one.

	Favourable	Non-Favourable	Total
onventional	40	70	110
New	60	30	90
Total	100	100	N=200

(b) The height of six randomly chosen sailors are (in inches): 72, 71, 69, 68, 65, 63. Those of ten randomly chosen army men are 73, 72, 71, 70, 69, 69, 66, 65, 62, 61. Analyse the concept that the army men are shorter, on the average, than sailors.

(or)

20 .(a) Test whether the sample having the values 63, 63, 64, 55, 69, 70, 70 and 71 has been chosen from a population with mean of 65 at 5% level of significance.

(b) Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables.

Sample I	18	13	12	15	12	14	16	14	15
Sample II	16	19	13	16	18	13	15		

Do the estimates of the population variance differ significantly at 5% level?

SOLVED UNIVERSITY QUESTION PAPERS

B.Tech. DEGREE EXAMINATION, MAY 2016
Fourth Semester
MATHEMATICS IV
(2013-2014 Batch onwards)

Time: Three hours **Maximum: 75 marks**

Section A-(10X2=20 marks)
Answer all the questions
All questions carry equal marks

1. From the partial differential equation from $z = f(2x-6y)$

$z = f(2x - 6y)$

$p = \frac{\partial z}{dx} = 2f' \Rightarrow \frac{p}{2} = f'$

$q = \frac{\partial z}{dy} = -6f' \Rightarrow \frac{q}{-6} = f'$

$\therefore \frac{p}{2} = \frac{q}{-6}$

The required PDE is $3p + q = 0$

2. **Find the complete integral of** $z = px + qy + p^2 - q^2$ **.**

Given $z = px + qy + p^2 - q^2 \text{-------} \rightarrow (1)$

The equation is of the form z=px+qy+f(p,q)

W. k. t p = a and q = b

$z = ax + by + a^2 - b^2 \text{-------} \rightarrow (2)$

This is the complete solution.

3. **Write all possible solutions of one dimensional wave equation.**

$(i)\quad y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

$(ii)\ y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos pat + c_8 \sin pat)$

$(iii)\ y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$ are the possible solutions of the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

4. **What are the assumptions made in deriving the one dimensional wave equation?**

1. The motion takes place entirely in one plane.
2. The effect of the friction is negligible
3. The string is perfectly flexible
4. TensionT in the stringto be so large that gravity may be negelected in comparison with it.

5. **A rod of 30 cm long its ends A and B kept at** $20^\circ c$ **and** $80^\circ c$ **until steady state conditions prevails.Define steady state temperature distribution.**

The one dimensional heat equation is $\frac{\partial^2 u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

Correct solution is u(x) = ax+b …..(1)

With boundary conditions are

(i) u(0) = 20

(ii) u(l) = 80

Subst (i) in (1) u(0) = b= 20, u(x) = ax+20

Subst (ii) u(l) = al +20 = 80 , al = 80 – 20

al =60 , a = 60/l

u(x) = 2x+20.

6. **Give the three possible solutions of two dimensional steady state heat flow equation**.

(i) $u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

(ii) $u(x,y) = (c_5 \cos px + c_6 \sin px)(c_7 e^{py} + c_8 e^{-py})$

(iii) $u(x,y) = (c_9 x + c_{10})(c_{11} y + c_{12})$

7. **Define null hypothesis and alternate hypothesis**.

Null hypothesis

It is a definite statement about the population parameter which is usually a hypothesis of no difference and is denoted by H_0.

In case of signal statistic, H_0 will be that the sample statistic does not differ significantly from the hypothetical parameter value.

In case of two statistics, H_0 will be that the sample statistics do not differ significantly.

Alternative hypothesis

Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, usually denoted by H_1

8. Mention the various steps involved in testing of hypothesis.

1.Formulate H_0 and H_1

2.Choose level of significance α

3.Compute test statistic Z, using the data available in the problem.

4Pick out the critical value at α% level say Z_α

5.Draw conclusion

If $|Z| < Z_\alpha$, accept H_0 at α% level

Otherwise, reject H_0 at α% level.

9. Write any two applications of t-test.

It is used

(i) To test the significance of the difference between two sample means

(ii) To test the significance of the coefficient of correlation.

10. Define ψ^2 – test for goodness of fit.

If O_i (i = 1, 2, ... n) is a set of observed (experimental) frequencies and E_i (i = 1, 2, . . . n) is the corresponding set of expected (theoretical) frequencies, then the statistic ψ^2 is defined as $\psi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ with d.o.f = n - 1

Part B (5x11=55)

Answer ONE full question from each unit

UNIT I

11. Solve the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

Solution:

The equation is of the form pP + qQ = R

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \lambda$

Here P= $x^2(y-z)$,Q= $y^2(z-x)$,R= $z^2(x-y)$

$$\therefore \frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} = \lambda$$

Case-1:

Choose the multiplier $\ell = \frac{1}{x}, \; m = \frac{1}{y}, \; n = \frac{1}{z}$

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{xy - xz + yz - yx + zx - zy} = \lambda$$

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

Integrating,

$$\log x + \log y + \log z = \log c_1$$

$xyz = c_1$.

Case-2:

Choose the multiplier $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$

$$\frac{\frac{dx}{x^2}+\frac{dy}{y^2}+\frac{dz}{z^2}}{y - z + z - x + x - y} = \lambda$$

Integrating,

$$\int \frac{dx}{x^2} + \int \frac{dy}{y^2} + \int \frac{dz}{z^2} = 0$$

$$\boxed{c_2 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

$\therefore$ The general solution is, $\boxed{\varphi(xyz, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}) = 0}$

(or)

12. **Solve** $(D^2 + 4DD' - 5D'^2)z = 3e^{2x-y} + \sin(x-2y)$.

Solution:

To Find CF:

$Put\ D = m$ and $D' = 1$

$m^2 + 4m - 5 = 0$

$(m-1)(m+5) = 0$

$m = 1, -5$

$$\boxed{C.F = f_1(y+x) + xf_2(y-5x)}$$

To Find PI_1 :

$$PI_1 = \frac{1}{D^2 + 4DD' - 5D'^2} 3e^{2x-y}$$

$Put\ D = 2, \quad and \quad D' = -1$

$$= \frac{1}{4-8-5} 3e^{2x-y}$$

$$= -\frac{1}{3} e^{2x-y}$$

To find PI2:

$$PI_2 = \frac{1}{D^2 + 4DD' - 5D'^2} \sin(x-2y)$$

$Put\ D^2 = -1, \quad DD' = 2 \quad and \quad D'^2 = -4$

$$= \frac{1}{-1+8+20} \sin(x-2y)$$

$$= \frac{1}{27} \sin(x-2y)$$

The general solution is $z = CF + PI_1 + PI_2$

$$z = f_1(y+x) + xf_2(y-5x) - \frac{1}{3}e^{2x-y} + \frac{1}{27}\sin(x-2y)$$

UNIT II

13.A string is stretched and fastened to two points *l* apart. Motion is started by displacing the string into the form $k(lx-x^2)$ from which it is released at time t=0. Find the displacement of any point on the string at a distance from one end at time 't'.

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

From the given problem, we get the following boundary conditions

(i) $y(0,t) = 0$ for all $t > 0$

(ii) $y(l,t) = 0$ for all $t > 0$

(iii) $\frac{\partial y}{\partial t}(x,0) = 0$ (initial velocity is zero),0 < x < l

(iv) $y(x,0) = k(l\,x - x^2)$, 0 < x < l

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \ldots\ldots..(1)$$

Applying the condition (i) **in (1) we get**

$y(0,t) = A(C\cos pat + D\sin pat) = 0$

i.e.., $(C\cos pat + D\sin pat) \neq 0$

$\therefore A = 0$

Substitute $A = 0$ in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \ldots\ldots(2)$$

Appling condition (ii) **in (2) we get**

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$\sin pl = 0$

$pl = \sin^{-1}(0)$

$i.e..\ \ pl = n\pi \quad (\sin n\pi = 0)$

$i.e\ p = \frac{n\pi}{l}$ (n being an integer)

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left[C\cos\left(\frac{n\pi at}{l}\right) + D\sin\left(\frac{n\pi at}{l}\right)\right] \ldots\ldots\ldots\ldots\ldots(3)$$

Before applyingcondition (iii), differentiating (3) partially w.r.t't'

$$\frac{\partial y}{\partial t}(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left(-C\sin\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) + D\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right)\right) \text{------- (4)}$$

Applying condition (iii) in eqn (4), we get

$$\frac{\partial y}{\partial t}(x,0) = B\sin\left(\frac{n\pi x}{l}\right)\left(D\frac{n\pi a}{l}\right) = 0$$

Here $B\sin\frac{n\pi x}{l} \neq 0$ And $\frac{n\pi a}{l} \neq 0$

$\therefore \quad D = 0$

Therefore sub, D = 0 in (3) we get,

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)C\cos\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right) \text{---------- (5)}$$

Where $B_n = BC$

$\therefore$ The most general solutions of (5) can be written as

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Applying the boundary conditions (iv) in (6) we get

$$y(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = k\,(l\,x - x^2)$$

To find B_n expand $k\,(l\,x - x^2)$ in a half range fouriersine series in the interval $(0,l)$

$$\therefore \; B_n = \frac{2}{l}\int_0^l k(lx - x^2)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2k}{l}\left[(lx - x^2)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (l - 2x)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + (-2)\left(\frac{\cos\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right)\right]_0^{\ell}$$

$$= \frac{2k}{l}\left[0 - 2\left(\frac{\cos n\pi}{\frac{n^3\pi^3}{l^3}}\right) - 0 + 2\left(\frac{\cos 0}{\frac{n^3\pi^3}{l^3}}\right)\right] \qquad [\therefore \quad \sin n\pi = 0]$$

$$= \frac{2k}{\ell} \times \frac{2\ell^3}{n^3\pi^3}\left[-(-1)^n + 1\right]$$

$$B_n = \frac{4kl^2}{n^3\pi^3}\left[1 - (-1)^n\right]$$

$$B_n = \begin{cases} \frac{8kl^2}{n^3\pi^3}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

Substituting B_n in (6) we get

$$y(x,t) = \sum_{1,3,5..}^{\infty} \frac{8kl^2}{n^3\pi^3}\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)$$

(or)

14. A string is stretched between two fixed points at a distance apart and the points on the string are given initial velocity V, where $v=\begin{cases}\frac{c}{l}x\,, & 0<x<l\\ \frac{c}{l}(2l-x)\,, & l<x<2l\end{cases}$ **. Find the displacement .**

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$

Consider here $2l=L$

With the boundary conditions

$(i)\ y(0,t)=0,\ for\,all\,t>0$

$(ii)\ y(L,t)=0,\ for\,all\,t>0$

$(iii)\ y(x,0)=0,\ for\,all\,x$

$$(iv)\frac{\partial y(x,0)}{\partial t}=\begin{cases}\frac{2cx}{L} & ,\ 0<x<\frac{L}{2}\\ \frac{2c}{L}(L-x), & \frac{L}{2}<x<L\end{cases}$$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t)=(A\cos px+B\sin px)(C\cos pat+D\sin pat) \qquad \text{…….(1)}$$

Applying the condition (i) **in (1) we get**

$$y(0,t)=A(C\cos pat+D\sin pat)=0$$

But $(C\cos pat+D\sin pat)\neq 0$

$$\therefore A=0$$

Substituting in (1) we get

$$y(x,t)=B\sin px(C\cos pat+D\sin pat) \qquad \text{……..(2)}$$

Now applying condition (ii) **in (2) we get**

$$y(\ell,t)=B\sin pl(C\cos pat+D\sin pat)=0$$

Here $C\cos pat+D\sin pat\neq 0$ and $B\neq 0$ (therefore it is defined for all t)

Therefore we take $\sin pl=0$

$$pl=\sin^{-1}(0)$$

$i.e..\ \ pl=n\pi \quad (\sin n\pi=0)$

$i.e\ \ p=\frac{n\pi}{l}$ (n being an integer)

Now substituting $p=\frac{n\pi}{l}$ in (2) we get

$$y(x,t)=B\sin\frac{n\pi x}{l}\left[C\cos\frac{n\pi at}{l}+D\sin\frac{n\pi at}{l}\right] \qquad \text{…….(3)}$$

Applying condition (iii) in (3) we get

$$y(x,0) = B\sin\frac{n\pi x}{l}.C = 0$$

But $B \neq 0$ *and* $\sin\frac{n\pi x}{l} \neq 0$

$\therefore C = 0$

Substituting in (3) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)D\sin\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n\sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right) \text{ ---------- (4)} \quad \text{where } B_n = BD$$

The most general solution equation is

$$y(x,t) = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{L}\sin\frac{n\pi at}{L}$$

Diff w.r.t. t we get

$$\frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{\infty} c_n \frac{n\pi a}{L}\sin\frac{n\pi x}{L}\cos\frac{n\pi at}{L}$$ put t=0 we get

$$\frac{\partial y(x,0)}{\partial t} = \sum_{n=1}^{\infty} c_n \frac{n\pi a}{L}\sin\frac{n\pi x}{L} = f(x) \text{ where } f(x) = \begin{cases} \frac{2cx}{L} & , \ 0 < x < \frac{L}{2} \\ \frac{2c}{L}(L-x), & \frac{L}{2} < x < L \end{cases}$$

To find c_n expand $f(x)$ in a half range Fourier sine series $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

Comparing we get $\sum_{n=1}^{\infty} c_n \frac{n\pi a}{L}\sin\frac{n\pi x}{L} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

$$c_n \frac{n\pi a}{L} = b_n, \ c_n = \frac{b_n L}{n\pi a}$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$= \frac{2}{L}\left[\int_0^{\frac{L}{2}} \frac{2cx}{L}\sin\frac{n\pi x}{L}dx + \int_{\frac{L}{2}}^{L} \frac{2c}{L}(L-x)\sin\frac{n\pi x}{L}dx\right]$$

$$= \frac{4c}{L^2}\left[\left\{x\left(\frac{-\cos\frac{n\pi x}{L}}{\frac{n\pi}{L}}\right) - (1)\left(\frac{-\sin\frac{n\pi x}{L}}{\frac{n^2\pi^2}{L^2}}\right)\right\}_0^{\frac{L}{2}} + \left\{(L-x)\left(\frac{-\cos\frac{n\pi x}{L}}{\frac{n\pi}{L}}\right) - (-1)\left(\frac{-\sin\frac{n\pi x}{L}}{\frac{n^2\pi^2}{L^2}}\right)\right\}_{\frac{L}{2}}^{L}\right]$$

$$= \frac{4c}{L^2}\left[-\frac{\frac{L}{2}\cos\frac{n\pi}{2}}{\frac{n\pi}{L}} + \frac{\sin\frac{n\pi}{2}}{\frac{n^2\pi^2}{L^2}} + \frac{\frac{L}{2}\cos\frac{n\pi}{2}}{\frac{n\pi}{L}} + \frac{\sin\frac{n\pi}{2}}{\frac{n^2\pi^2}{L^2}}\right]$$

$$= \frac{4c}{L^2}\ \frac{2\sin\frac{n\pi}{2}}{\frac{n^2\pi^2}{L^2}} = 8c\frac{\sin\frac{n\pi}{2}}{n^2\pi^2}$$

$$b_n = 8c\frac{\sin\frac{n\pi}{2}}{n^2\pi^2}$$

Substuting b_n we get $c_n = \dfrac{8cL\sin\frac{n\pi}{2}}{n^3\pi^3 a}$

Replacing L by $2l$ we get $c_n = \dfrac{16cl\sin\frac{n\pi}{2}}{n^3\pi^3 a}$

Substuting in most general solution we get

$$y(x,t) = \sum_{n=1}^{\infty}\frac{16cl\sin\frac{n\pi}{2}}{n^3\pi^3 a}\sin\frac{n\pi x}{2l}\sin\frac{n\pi at}{2l}.$$

UNIT III

15. A metallic bar10 cm long with insulated sides has its ends A and B kept at 2 $0^\circ c$ **and** $40^\circ c$ **respectively, until steady state conditions prevail. The temperature atA is then suddenly raised to** $50^\circ c$ **and B is lowered to** $10^\circ c$**. Find the subsequent temperature distribution.**

Solution:

Consider State I x = 20 ,u = 20 and u = 40.

Consider State II x = 50 ,u = 20 and u = 10 and l =10.

The one dimensional heat equation is $\frac{\partial^2 u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

Here there are two states

$i.e \quad u(x,t) = u_1(x) + u_2(x,t) \; ------\rightarrow (1)$

Steady State I	Steady State II
$x = 0 \qquad x = 20$ $u = 30 \qquad u = 80$ The steady state solution is $u(x,0) = \left(\frac{\theta_2 - \theta_1}{\ell}\right)x + \theta_1$ Here $\theta_1 = 30, \quad \theta_2 = 80 \;\; and \;\; \ell = 20$ $u(x,0) = \left(\frac{80-30}{20}\right)x + 30$ $u(x,0) = \frac{50}{20}x + 30$ $u(x,0) = \frac{5}{2}x + 30$ Now the B.C Are 1) $u(0,t) = 40^{\circ}c, \forall t > 0$ 2) $u(30,t) = 60^{\circ}c, \forall t > 0$ 3) $u(x,0) = \frac{5}{2}x + 30, \quad 0 < x < 20$	$x = 0 \qquad x = 20$ $u = 40 \qquad u = 60$ The steady state solution is $u(x,0) = \left(\frac{\theta_2 - \theta_1}{\ell}\right)x + \theta_1$ Here $\theta_1 = 40, \quad \theta_2 = 60 \;\; and \;\; \ell = 20$ $u_1(x) = \left(\frac{\theta_2 - \theta_1}{l}\right)x + \theta_1$ $= \frac{60-40}{20}x + 40$ $u_1(x) = x + 40$ Now the B.C Are $(i) \;\; u_1(0) = 40^{\circ}c$ $(ii) \;\; u_1(30) = 60^{\circ}c$ $(iii) \; u_1(x) = x + 40$

To **find** $u_2(x,t)$ **:**

But $\quad u_2(x,t) = u(x,t) - u_1(x)$

$(i) \;\; u_2(0,t) = u(0,t) - u_1(0) \;=\; 40^{\circ} - 40^{\circ} = 0$

$(ii) \;\; u_2(20,t) = u(20,t) - u_1(20) \;= 60^{\circ} - 60^{\circ} = 0$

$(iii) \;\; u_2(x,0) = u(x,0) - u_1(x) \;=\; \frac{5}{2}x + 30 - x - 40$

$\therefore \; u_2(x,t) = \frac{3}{2}x - 10$

Now the boundary condition are

$i) \;\; u_2(0,t) = 0^{\circ}c, \quad \forall t > 0$

$ii) \;\; u_2(20,t) = 0^{\circ}c, \quad \forall t > 0$

$iii) \;\; u_2(x,0) = \frac{3}{2}x - 10, \quad 0 < x < 20$

The suitable solution are,

$$u_2(x,t) = [A\cos px + B\sin px]Ce^{-\alpha^2 p^2 t} \quad ------\rightarrow (2)$$

Apply (i) boundary condition in (2)

$u_2(0,t) = [A\cos 0 + B\sin 0]Ce^{-\alpha^2 p^2 t}$

$i.e) \quad ACe^{-\alpha^2 p^2 t} = 0$

But $\qquad Ce^{-\alpha^2 p^2 t} \neq 0$

$\therefore \quad A = 0$

Substitute in equation (2)

$u_2(x,t) = B\sin pxCe^{-\alpha^2 p^2 t} \quad -----\rightarrow (3)$

Apply (ii) boundary condition in (3)

$u_2(20,t) = B\sin 20p\ Ce^{-\alpha^2 p^2 t} = 0$

$Since \quad B \neq 0 \ \ and \quad Ce^{-\alpha^2 p^2 t} \neq 0$

$\therefore \ \sin 20p = 0$

$20p = \sin^{-1}(0)$

$20p = n\pi$

$p = n\pi / 20$

Substitute in equation (3)

$$u_2(x,t) = B\sin\left(\frac{n\pi x}{20}\right)Ce^{-\alpha^2 \frac{n^2\pi^2}{20^2}t}$$

$$u_2(x,t) = B_n \sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2 \frac{n^2\pi^2}{20^2}t}$$

The most general solution can be written as

$$\boxed{u_2(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2 \frac{n^2\pi^2}{20^2}t}} \quad -------\rightarrow (4)$$

Apply (iii) boundary condition in (4)

$$u_2(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{20}xe^{-\alpha^2 \frac{n^2\pi^2}{20^2}(0)} = f(x)$$

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{20}\right) = \frac{3}{2}x - 10$$

To find B_n :

$$B_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$$

$$B_n = \frac{2}{20}\int_0^{20}\left(\frac{3}{2}x - 10\right)\sin\left(\frac{n\pi x}{20}\right)dx$$

$$= \frac{1}{10}\left[\left(\frac{3}{2}x - 10\right)\left(\frac{-\cos\left(\frac{n\pi x}{20}\right)}{\frac{n\pi}{20}}\right) - \left(\frac{3}{2}\right)\left(\frac{-\sin\left(\frac{n\pi x}{20}\right)}{\frac{n^2\pi^2}{20^2}}\right)\right]_0^{20}$$

$$= \frac{1}{10}\left[\frac{-20(20)}{n\pi}\cos(n\pi) + \frac{(-20)(10)}{n\pi}\cos 0\right]$$

$$= \frac{1}{10} \times \frac{-(20)(10)}{n\pi}\left[2(-1)^n + 1\right]$$

$$= \frac{-20}{n\pi}\left[2(-1)^n + 1\right]$$

Substitute in equation (3)

$$u_2(x,t) = \frac{-20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[2(-1)^n + 1\right]\sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2\frac{n^2\pi^2}{400}t}$$

But $\quad u(x,t) = u_1(x) + u_2(x,t)$

$$u(x,t) = x + 40 - \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[2(-1)^n + 1\right]\sin\left(\frac{n\pi x}{20}\right)e^{-\alpha^2\frac{n^2\pi^2}{400}t}$$

(or)

16. **A rectangular plate with insulated surface is10 cm wide and so long when compared to its width that it may be considered infinite in length. The temperature along the short edge y=0 is given by** $u(x,0) = \begin{cases} 20x, & 0 < x < 5, \\ 20(10-x), & 5 < y < 10. \end{cases}$ **The other short edge are kept at** 0^0C**, find the steady state temperature distribution in the plate.**

Solution:

Let u(x,y) be the temperature at any point (x,y). Then u(x,y) satisfies the Laplace equation

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} = 0$ with the boundary conditions

$(i)\, u(0,y) = 0 \qquad (ii)\, u(10,y) = 0$

$(iii)\, u(x,\infty) = 0 \qquad (iv)\, u(x,0) = \begin{cases} 20x & ,0 \le x \le 5 \\ 20(10-x) & ,5 \le x \le 10 \end{cases}$

The correct solution equation is given by

$u(x,y) = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py})$ --------- (1)

Appiling (i) in (1) we get c_1=0.Subst c_1=0 we get

$u(x,y) = (c_2 \sin px)(c_3 e^{py} + c_4 e^{-py})$ --------(2) Subst (ii) we get

$u(10,y) = (c_2 \sin p10)(c_3 e^{py} + c_4 e^{-py}) = 0$

$p = \frac{n\pi}{10}$

Subst $p = \frac{n\pi}{10}$ in (2) we get

$u(x,y) = c_2 \sin\frac{n\pi x}{10}\left(c_3 e^{\frac{n\pi y}{10}} + c_4 e^{-\frac{n\pi y}{10}}\right)$ ----------(3)

Subst (iii) in (3)we get

$u(x,\infty) = c_2 \sin\frac{n\pi x}{10}\left(c_3 e^{\frac{n\pi y}{10}} + c_4 e^{-\frac{n\pi y}{10}}\right) = 0$, c_3 =0

Subst c_3 =0, we get

$$u(x,y)=c_2c_4\,e^{-\frac{n\pi y}{10}}\sin\frac{n\pi x}{10}$$

$$=c_n\,e^{-\frac{n\pi y}{10}}\sin\frac{n\pi x}{10}$$

The most general solution equation is $u(x,y)=\sum c_n\,e^{-\frac{n\pi y}{10}}\sin\frac{n\pi x}{10}$ --------(4)

Applying (iv) condition we get

$$u(x,0)=\sum_{n=1}^{\infty}c_n\sin\frac{n\pi x}{10}=f(x)$$

$$=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{10}=f(x)$$

$$\therefore b_n=c_n$$

Where $b_n=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx=\frac{2}{10}\int_0^{10}f(x)\sin\frac{n\pi x}{10}dx$

$$=\frac{2}{10}\int_0^5 20x\sin\frac{n\pi x}{10}dx+\frac{2}{10}\int_5^{10}20(10-x)\sin\frac{n\pi x}{10}dx$$

$$=\frac{800}{n^2\pi^2}\sin\frac{n\pi}{2}$$

Subst b_n in (4) we get $u(x,y)=\sum_{n=1}^{\infty}\frac{800}{n^2\pi^2}e^{-\frac{n\pi y}{10}}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{10}$

UNIT IV

17. **Fit a parabola $y=ax^2+bx+c$ in least squares sense to the data:**

X:	**10**	**12**	**15**	**23**	**20**
Y:	**14**	**17**	**23**	**25**	**21**

Let, the parabola equation be,

$y=ax^2+bx+c$ ----------(1)

The normal equations are,

$\sum x^2y=a\sum x^4+b\sum x^3+c\sum x^2$ ----------(2)

$\sum xy=a\sum x^3+b\sum x^2+c\sum x$ ----------(3)

$\sum y=a\sum x^2+b\sum x+nc$ ----------(4)

x	**y**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
10	14	100	1000	10000	140	1400
12	17	144	1728	20736	204	2448
15	23	225	3375	50625	345	5175
23	25	529	12167	279841	575	13225
20	21	400	8000	160000	420	8400
$\sum x = 80$	**$\sum y = 100$**	**$\sum x^2 = 1398$**	**$\sum x^3 = 26270$**	**$\sum x^4 = 521202$**	**$\sum xy = 1684$**	**$\sum x^2y = 30648$**

Substitute in eqn (1),(2) and (3)

30648=521202a + 26270b + 1398c

1684=26270a + 1398b + 80c

100=1398a + 80b + 5c

Solving this we get

a = - 0.0695, b = 3.00, c = -8.728

Sub a,b,c values in y =ax^2+bx+c

$$y = -0.0695x^2+3.00x - 8.728$$

(or)

18. **A machine produced 20 defective units in a sample of 400. After overhauling the machine it produced 10 defective units in a batch of 300. Has the machine improved in production due to Overhauling? Test it a 5% L.O.S.**

Step 1:

n_1 =400	n_2 =300
$X_1 = 20$	$X_2 = 10$
p_1 =20/400	p_2 =10/300
p_1 =0.05	p_2 =0.033

Step 2:

Null hypothesis, H_0:

$p_1 = p_2$

Alternate hypothesis,H_1 :

$p_1 > p_2$ (Two tailed test)

Step 3: Test Statistic

$$z = \frac{|p_1 - p_2|}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Where

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

=((400*0.05)+(300*0.033))/(400+300)

=0.0427

Q=1 – P=0.9573

$$Z=|0.05-0.033| / \sqrt{(0.0427*0.9573)\left(\frac{1}{400}+\frac{1}{300}\right)}$$

Z = 1.103

Step 4:

At 5% level of significance, Z=1.96

∴calculated value <Tabulated value

i.eH_0 is accepted.

Conclusion:

∴ The proportion of men and women in favour of proposals are same.

UNIT V

19. A random sample of 10 boys had the following I.Q's70,120,110,101,88,83,95,107,100. Do these data support the assumption of population mean I.Q of 100?.

Solution:

Step1:

Given n = 10

$\mu = 100$

$\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$

Here S.D. and mean of sample is not given directly. We have to determine these S.D. and mean as follows:

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.48
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
970		**1833.60**

Mean $\bar{x} = \sum \frac{x}{n} = \frac{972}{10} = 97.2$

We know that $s^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n} = \frac{1833.60}{10} = 183.36$

Therefore, Standard Deviation (s) = $\sqrt{183.36} = 13.541$

Step 2:

Null Hypothesis H_0= μ = 100The data support the assumption of a population mean I.Q. of 100 in the population.

Alternative Hypothesis $H_1 : \mu \neq 100$

Step 3: The test statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$ (Note that S.D. is not given directly)

$$t = \frac{97.2 - 100}{\frac{13.541}{\sqrt{9}}} = -0.6203$$

|t| = 0.6203

Step 4:

The table value of t at 5% level of significance for 9d.f. for two tailed test is 2.26
Since calculated t value<tabulated t value
we accept the null hypothesis H_0.

Conclusion:

The data support the assumption of mean I.Q. of 100 in the population.

(or)

20 . The following table gives the number of aircraft accidents that occur during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Total**
No. of accidents	**14**	**18**	**12**	**11**	**15**	**14**	**84**

Solution:

Step 1:

Given

n = 6

Step 2:

Null hypothesis H_0 : There are no significance difference between O_i and E_i

Step 3:

$$E = \frac{\sum o}{n} = 84/6=14$$

$$\chi^2 = \sum \left[\frac{(O-E)^2}{E} \right]$$

O	E	$O-E$	$\frac{(O_i - E_i)^2}{E_i}$
14	14	0	0
18	14	4	1.143
12	14	-2	0.286
11	14	-3	0.643
15	14	1	0.071
14	14	0	0

$$\chi^2 = \sum \left[\frac{(O-E)^2}{E} \right] = 2.143$$

Step 4:

DOF= n – 1 = 6 – 1=5

At 5% $\Psi^2{}_{tab} = 11.07$

$\Psi^2{}_{ca;} < \Psi^2{}_{tab}$. H_0 is accepted.

Conclusion:

There are no significance difference between O_i and E_i

5604064

B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016

Fourth Semester

MATHEMATICS IV

(From 2013 – 14 onwords)

Time: Three hours **Maximum: 75 marks**

Section A-(10X2=20 marks)

Answer all the questions

All questions carry equal marks

1.Form partial differential equation by eliminating the arbitrary constants from $z=(x^2+a)(y^2+b)$.

Given: PDE **$z = (x^2 + a)(y^2 + b)$----------(1)**

Diff w.r.to 'x'

$$\frac{\partial z}{\partial x} = 2x\left(y^2 + b\right)$$

$$\left(y^2 + b\right) = \frac{p}{2x} \text{--------} \rightarrow (2)$$

Diff w.r.to 'y'

$$\frac{\partial z}{\partial y} = 2y\left(x^2 + a\right)$$

$$\left(x^2 + a\right) = \frac{q}{2y} \text{---------} \rightarrow (3)$$

Substitute eqn (2) and (3) in (1)

$$z = \frac{p}{2x} \times \frac{q}{2y}$$

$$z = \frac{pq}{4xy}$$

$$4xyz = pq$$

2.Solve $p^2 + q^2 = npq$

The given equation is of the form F(p,q)=0

Given: $p^2 + q^2 = npq \text{------} 1$

Let us assume $z = ax + by + c \text{--------} 2$

$D.p.w.r.to\, x \,\&\, y \;in \text{--} 2$

$$\frac{\partial z}{\partial x} = a \qquad \text{and} \qquad \frac{\partial z}{\partial y} = b$$

$$p = a \qquad \text{and} \qquad q = b$$

Sub in (2)

Sub p and q values in----1

$$a^2 + b^2 = nab$$

$$b^2 - nab + a^2 = 0$$

$$b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{na \pm \sqrt{n^2a^2 - 4a^2}}{2}$$

$$b = \frac{na \pm \sqrt{a^2(n^2 - 4)}}{2}$$

$$b = \frac{a}{2}(n \pm \sqrt{n^2 - 4})$$

Sub in (2) $z = ax + a/2(n \pm \sqrt{n^2 - 4})y + c$

This is the complete solution.

3.State the assumptions made while deriving the equation of Vibration of a string in the simple form.

1. The motion takes place entirely in one plane.
2. The effect of the friction is negligible
3. The string is perfectly flexible
4. TensionT in the stringto be so large that gravity may be negelected in comparison with it.

4.What are the possible solutions of one dimensional wave equation?

(i) $y(x,t) = (c_1e^{px} + c_2e^{-px})(c_3e^{pat} + c_4e^{-pat})$

(ii) $y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos pat + c_8 \sin pat)$

(iii) $y(x,t) = (c_9x + c_{10})(c_{11}t + c_{12})$ are the possible solutions of the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$.

5. What does α^2 represent in the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$?

The one dimensional heat flow equation is given by $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ where $\alpha^2 = \frac{k}{c\rho}$ is called diffusivity of the material of the body through which heat flows. Here ρ - density, c-specific heat, k- thermal conductivity.

6. State the two laws of thermodynamics used in the derivation of two dimensional heat flow equations.

(i). The sides of the bar are insulated so that the loss or gain of heat from the sides by conduction or radiation is negligible.

(ii). The same amount of heat is applied at all points of the face.

7.What is the principle of least square?.

Let (x_i,y_i) ,i=1,2,…n be n sets of observations and let y = f(x). Let the residual d_i= $y_i - f(x_i)$, E=$\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n}(x_i - f(x_i))^2$ is the sum of the squares of the residuals. If E=0, then all the n points will lie on y = f(x). Otherwise, we choose f(x) such that E is minimum. This method is called method of least squares.

8.Explain type I and type II errors.

Type I error : Reject H_0 when it is true

Type II error : Accept H_0 when it is wrong

9. What are the assumptions made in the student's t-test?

(i) The parent population from which the sample drawn is normal.

(ii) The sample observations are independent. (i.e.) the sample is random.

(iii) The population S.D. σ is unknown.

10. State any two important applications of ψ^2 distribution.

ψ^2 – test is used to test

(i) The independence of attributes.

(ii) The goodness of fit.

(iii) Whether two or more attributes are associated or not.

(iv).Whether the theoretical distributions such as Binomial, Poisson, Normal etc., fit empirical distributions.

Section B-(5X11=55 marks)

UNIT I

11.(a) Find the singular integral of z = px+qy+ (p/q- p).

Solution:

The given equation is of the form clairaut's equation, z = px+qy+(p/q-p)-------1

Put p=a,q=b

Z=ax+by+(a/b-a)----------2

This is the complete solution.

To find singular solution:

D,p. w.r.t 'a'

Diff, p. w.r.t 'a'

$$0 = x + \frac{1}{b} - 1$$

$$b = \frac{1}{1-x}.$$

Diff, p. w.r.t 'b'

$$0 = y - \frac{a}{b^2} \quad , \quad a = yb^2, \quad a = \frac{y}{(1-x)^2}.$$

Subt 'a' and 'b' in (2)

We get $z = \frac{y}{1-x}$.

(b) Solve $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$.

Solution:

The equation is of the form pP + qQ = R

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \lambda$

Here P= $x(y^2+z)$,Q= $y(x^2+z)$,R= $z(x^2-y^2)$

$$\frac{dx}{x\left(y^2+z\right)}=\frac{dy}{y\left(x^2+z\right)}=\frac{dz}{z\left(x^2-y^2\right)}$$

Consider $\frac{1}{x},\frac{1}{y},\frac{1}{z}$, we get

$$\frac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{\left(y^2+z\right)-\left(x^2+z\right)+\left(x^2-y^2\right)}=0\ ,\ \frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz=0$$

$$\log x+\log y+\log z=\log c_1$$

$$xyz=c_1.$$

Consider $xdx+ydy-dz$, $\frac{xdx+ydy-dz}{x^2\left(y^2+z\right)-y^2\left(x^2+z\right)-z\left(x^2-y^2\right)}=0$

$$x^2+y^2-2z=c_2.$$

$$\phi\left(xyz,x^2+y^2-2z\right)=0.$$

(or)

12. Solve $\left[D^3-7DD'^2-6D'^3\right]z=x^2y+\sin(x+2y)$

Solution:

To find CF:

$$Put\ D=m\ \ and\ \ D'=1$$

$$m^3-7m^2-6=0$$

$$m=-1\ and\ \ m^2-m-6=0$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & 0 & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$m=-1\ and\ \ (m-3)(m+2)=0$$

$$m=-1,-2,3$$

$$\therefore\ \boxed{C.F=f_1(y-x)+f_2(y-2x)+f_3(y+3x)}$$

To Find PI₁:

$$PI=\frac{1}{D^3-7DD'^2-6D'^3}x^2y$$

$$=\frac{1}{D^3\left[1-\left(\frac{7DD'^2}{D^3}-\frac{6D'^3}{D^3}\right)\right]}(x^2y)$$

$$=\frac{1}{D^3}\left[1-\left(\frac{7D'^2}{D^2}+\frac{6D'^3}{D^3}\right)\right]^{-1}(x^2y)$$

$$=\frac{1}{D^3}\left[1+\frac{7D'^2}{D^2}+\frac{6D'^3}{D^3}\right](x^2y)$$

$$=\frac{1}{D^3}\left[(1)x^2y\right]\quad \sin ce\ \ D'^2\ \ and\ \ D'^3\ \ are\ \ zero$$

$$= \frac{x^2 y}{D^3}$$

$$= \iiint x^2 y \; dx \; dx \; dx$$

$$= \iint \frac{x^3 y}{3} dx \; dx$$

$$= \int \frac{x^4 y}{12} dx$$

$$PI_1 = \frac{x^5 y}{60}$$

To find PI2:

$$PI_2 = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \sin(x+2y)$$

$$= \frac{1}{D(D^2) - 7DD'(D') - 6D'^2(D')} \sin(x+2y)$$

$$Put \; D^2 = -1, \quad DD' = -2 \quad and \quad D'^2 = -4$$

$$= \frac{1}{D(-1) - 7(-2)D' - 6(-4)D'} \sin(x+2y)$$

$$= \frac{1}{-D + 14D' + 24D'} \sin(x+2y)$$

$$= \frac{1}{38D' - D} \sin(x+2y)$$

$$= \frac{(38D' + D)}{(38D' - D)(38D' + D)} \sin(x+2y)$$

$$= \frac{(38D' + D)}{1444D'^2 - D^2} \sin(x+2y)$$

$$Put \; \mathrm{D}^2 = a^2 = -1 \; and \; D'^2 = b^2 = -4$$

$$= \frac{38D'(\sin(x+2y)) + D(\sin(x+2y))}{1444(-4)+1}$$

$$= \frac{38\cos(x+2y)(2) + \cos(x+2y)(1)}{-5776+1}$$

$$= \frac{76\cos(x+2y) + \cos(x+2y)}{-5775}$$

$$= -\frac{77\cos(x+2y)}{5775}$$

The general solution is $z = CF + PI_1 + PI_2$

$$\mathrm{z} = f_1(y-x) + f_2(y-2x) + f_3(y+3x) + \frac{x^5 y}{60} - \frac{77\cos(x+2y)}{5775}$$

UNIT II

3. A tightly stretched string has its ends fixed at x =0 and $x = l$. At time t =0 the initial displacement of the string is given by $f(x) = kx^2(l-x)$ where k is a constant and then released from rest. Find the displacement of any point x of the string at any time t.

Solution: The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

From the given problem, we get the following boundary conditions

(i) $y(0,t) = 0$ for all $t > 0$

(ii) $y(l,t) = 0$ for all $t > 0$

(iii) $\frac{\partial y}{\partial t}(x,0) = 0$ (initial velocity is zero), $0 < x < l$

(iv) $y(x,0) = kx^2(l-x)$, $0 < x < l$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \ldots\ldots..(1)$$

Applying the condition (i) **in (1) we get**

$y(0,t) = A(C\cos pat + D\sin pat) = 0$

i.e.., $(C\cos pat + D\sin pat) \neq 0$

$\therefore A = 0$

Substitute $A = 0$ in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \ldots\ldots(2)$$

Appling condition (ii) **in (2) we get**

$$y(l,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take

$\sin pl = 0$

$pl = \sin^{-1}(0)$

$i.e..\ \ pl = n\pi \quad (\sin n\pi = 0)$

$i.e\ \ p = \frac{n\pi}{l}$ (n being an integer)

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left[C\cos\left(\frac{n\pi at}{l}\right) + D\sin\left(\frac{n\pi at}{l}\right)\right] \ldots\ldots\ldots\ldots\ldots(3)$$

Before applyingcondition (iii), differentiating (3) partially w.r.t't'

$$\frac{\partial y}{\partial t}(x,t) = B\sin\left(\frac{n\pi x}{l}\right)\left(-C\sin\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) + D\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right)\right) \text{------- (4)}$$

Applying condition (iii) in eqn (4), we get

$$\frac{\partial y}{\partial t}(x,0) = B\sin\left(\frac{n\pi x}{l}\right)\left(D\frac{n\pi a}{l}\right) = 0$$

Here $B\sin\frac{n\pi x}{l} \neq 0$ And $\frac{n\pi a}{l} \neq 0$

$\therefore\ D = 0$

Therefore sub, D = 0 in (3) we get,

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)C\cos\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right) \text{---------- (5)}$$

Where $B_n = BC$

$\therefore$ The most general solutions of (5) can be written as

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}}$$

Applying the boundary conditions (iv) in (6) we get

$$y(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = kx^2(l-x)$$

To find B_n expand $kx^2(l-x)$ in a half range fouriersine series in the interval $(0,l)$

$$\therefore\ B_n = \frac{2}{l}\int_0^l kx^2(l-x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2k}{l}\left[\left(x^2l - x^3\right)\left(\sin\frac{n\pi x}{l}\right) - \left(2xl - 3x^2\right)\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + (2l-6x)\left(-\frac{\sin\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right) + 6\left(\frac{\cos\frac{n\pi x}{l}}{\frac{n^4\pi^4}{l^4}}\right)\right]_0^l$$

$$= \frac{2k}{l}\left[\frac{-l^2(-1)^n}{\frac{n^2\pi^2}{l^2}} + \frac{6(-1)^n}{\frac{n^4\pi^4}{l^4}} - \frac{6}{\frac{n^4\pi^4}{l^4}}\right] [\therefore\ \sin n\pi = 0]$$

$$= \frac{2k}{l}\times\frac{l^2}{n^2\pi^2}\left[-l^2(-1)^n + \frac{6l^2(-1)^n}{n^2\pi^2} - \frac{6l^2}{n^2\pi^2}\right]$$

$$= \frac{2k}{l}\times\frac{l^4}{n^2\pi^2}\left[(-1)^{n+1} + \frac{6(-1)^n}{n^2\pi^2} - \frac{6}{n^2\pi^2}\right]$$

$$= \frac{2kl^3}{n^2\pi^2}\left[(-1)^{n+1} + \frac{6(-1)^n}{n^2\pi^2} - \frac{6}{n^2\pi^2}\right]$$

Substituting B_n in (6) we get

$$y(x,t) = \sum_{n=1}^{\infty} \frac{2kl^3}{n^2\pi^2}\left[(-1)^{n+1} + \frac{6(-1)^n}{n^2\pi^2} - \frac{6}{n^2\pi^2}\right]\sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}$$

(or)

14. If a string of length 20 cm is initially at rest in its equilibrium position and each of its points is given a velocity v such that $v = \begin{cases} x & for\, 0 < x \le 10 \\ 20 - x & for\, 10 < x \le 20 \end{cases}$ **. Find the displacement of the string y(x,t).**

Solution: : The one dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

Put c=1 and $l = 20$

The boundary conditions are

$(i)\ y(0,t) = 0$ for all $t > 0$

$(ii)\ y(l,t) = 0$ for all $t > 0$

$(iii)\ y(x,0) = 0$, for all x in $(0,l)$ since the string has no displacement in its equilibrium Position.

$(iv)\ \frac{\partial y(x,0)}{\partial t} = \begin{cases} cx & 0 < x < l/2 \\ c(l-x) & l/2 < x < l \end{cases}$, for all x in $(0,l)$

Now the suitable solution which satisfies our boundary conditions is given by

$$y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \qquad \text{…….(1)}$$

Applying the condition (i) **in (1) we get**

$$y(0,t) = A(C\cos pat + D\sin pat) = 0$$

But $(C\cos pat + D\sin pat) \neq 0$

$$\therefore A = 0$$

Substituting in (1) we get

$$y(x,t) = B\sin px(C\cos pat + D\sin pat) \qquad \text{……..(2)}$$

Now applying condition (ii) **in (2) we get**

$$y(\ell,t) = B\sin pl(C\cos pat + D\sin pat) = 0$$

Here $C\cos pat + D\sin pat \neq 0$ and $B \neq 0$ (therefore it is defined for all t)

Therefore we take $\sin pl = 0$

$$pl = \sin^{-1}(0)$$

$i.e..\ pl = n\pi \quad (\sin n\pi = 0)$

$i.e\ p = \frac{n\pi}{l}$ (n being an integer)

Now substituting $p = \frac{n\pi}{l}$ in (2) we get

$$y(x,t) = B\sin\frac{n\pi x}{l}\left[C\cos\frac{n\pi at}{l} + D\sin\frac{n\pi at}{l}\right] \qquad \text{…….(3)}$$

Applying condition (iii) in (3) we get

$$y(x,0) = B\sin\frac{n\pi x}{l}.C = 0$$

But $B \neq 0$ *and* $\sin\frac{n\pi x}{l} \neq 0$

$$\therefore C = 0$$

Substituting in (3) we get

$$y(x,t) = B\sin\left(\frac{n\pi x}{l}\right)D\sin\left(\frac{n\pi at}{l}\right)$$

$$y(x,t) = B_n\sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right) \text{ ---------- (4)} \quad \text{where } B_n = BD$$

The most general equation of equation (4) is

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}} \qquad \text{.....(5)}$$

Before applying condition (iv), find $\frac{\partial y(x,t)}{\partial t}$ differentiating (5) partially w.r.t 't' we get

$$\frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)\left(\frac{n\pi a}{l}\right) \text{----------(6)}$$

Putting $t = 0$ in (6) we get

$$\frac{\partial y(x,0)}{\partial t} = \sum_{n=1}^{\infty} C_n\left(\frac{n\pi a}{l}\right)\sin\frac{n\pi x}{l} = \begin{cases} cx & 0 < x < l/2 \\ c(l-x) & l/2 < x < l \end{cases}, \text{ where } C_n = B_n\left(\frac{n\pi a}{\ell}\right)$$

Now to find 'B_n'expand the function in $f(x)$ in a half range Fourier sine series in the interval $0 < x < l$

Where $C_n = \frac{2}{l}\int_0^{\ell} f(x)\sin\frac{n\pi x}{l}\,dx$

$$= \frac{2}{\ell}\left(\int_0^{l/2} cx\sin\left(\frac{n\pi x}{l}\right)dx + \int_{l/2}^{l} c(l-x)\sin\left(\frac{n\pi x}{l}\right)dx\right)$$

$$= \frac{2c}{\ell}\left[x\left(-\frac{\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right) - \left(-\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^{l/2} + \left[(l-x)\left(-\frac{\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) + \left(-\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]$$

$$= \frac{2c}{l}\left[\frac{-l^2}{2n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \frac{l^2}{2n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{2c}{l}\left[\frac{2l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$C_n = \frac{4c\ell}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

But $\quad C_n = B_n\left(\frac{n\pi a}{\ell}\right)$

$$B_n = C_n\left(\frac{\ell}{n\pi a}\right)$$

$$B_n = \left\{\frac{4c\ell}{n^2\pi^2}\times\left(\frac{\ell}{n\pi a}\right)\sin\left(\frac{n\pi}{2}\right)\right.$$

$$B_n = \left\{\frac{4c\ell^2}{n^3\pi^3 a}\sin\left(\frac{n\pi}{2}\right)\right.$$

Substituting (6) in the most general solution we get

$$y(x,t) = \frac{4c\ell^2}{\pi^3 a}\sum_{n=1}^{\infty}\frac{1}{n^3}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{\ell}\right)\sin\left(\frac{n\pi at}{l}\right)$$

UNIT III

15. A rod of length '*l*' has its ends A and B kept at $0^\circ c$ and $120^\circ c$ respectively, until steady state conditions prevail. If the temperature of the end B is then suddenly reduced to $0^\circ c$ and kept so while that of A is maintained. Find the temperature distribution in the rod. (Dec 2012)

Solution: Consider here $q_1 = 0$ and $q_2 = 120$.

We know that the heat flow equation is $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$

Given $\theta_1 = 0$ *and* $\theta_2 = 200$

w.k.t $\quad u = \left(\frac{\theta_2 - \theta_1}{\ell}\right)x + \theta_1$

$$u = \left(\frac{200-0}{\ell}\right)x + 0$$

$$u = \frac{200}{\ell}x$$

Now the boundary conditions are

$(i)\ u(0,t) = 0,\quad$ for all $\quad t > 0$

$(ii)\ u(l,t) = 0,\quad$ for all $\quad t > 0$

$(iii)\ u(x,0) = \frac{200x}{l},\ 0 < x < \ell$

The suitable solution of $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ is

$$u(x,t)=(A\cos px+B\sin px)e^{-\alpha^2p^2t} \qquad \text{----(1)}$$

Applying condition (i) we get

$$u(0,t)=Ae^{-\alpha^2p^2t}=0$$

$$But\ \ e^{-\alpha^2p^2t}\neq 0 \qquad \therefore A=0$$

Substituting $A=0$ in (1) we get

$$u(x,t)=B\sin pxe^{-\alpha^2p^2t}=0 \qquad \text{----(2)}$$

Applying condition (ii) in (2) we get

$$u(l,t)=B\sin ple^{-\alpha^2p^2t}=0$$

Here $B\neq 0$ since if $B=0$ we get trivial solution.

$e^{-\alpha^2p^2t}\neq 0$ since it is true for all 't'

Hence $\sin pl=0$ i.e., $pl=n\pi$

Therefore $p=\frac{n\pi}{l}$

Substituting $p=\frac{n\pi}{l}$ in (2) we get

$$u(x,t)=B\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}$$

The most general solution can be written as

$$\boxed{u(x,t)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2n^2\pi^2t}{l^2}}} \qquad \text{----(3)}$$

Applying condition (iii) in (3) we get

$$u(x,0)=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}=\frac{200x}{l}$$

To find B_n expand $\frac{200x}{l}$ in $(0,l)$ in half- range Fourier sine series we get,

$$\frac{200x}{l}=\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{l}$$

Now $B_n=\frac{2}{l}\int_0^l\frac{200x}{l}\sin\frac{n\pi x}{l}dx$

$$=\frac{400}{l^2}\left[x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)-1\left(\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l=\frac{400}{l^2}\left[\frac{-l^2\cos n\pi}{n\pi}\right]$$

$$=\frac{400}{n\pi}(-1)^{n+1}\left[\because\cos n\pi=(-1)^n\right]$$

Substituting in (3) we get

$$u(x,t)=\sum_{n=1}^{\infty}\frac{400}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{l}e^{-\frac{\alpha^2 n^2\pi^2 t}{l^2}}$$

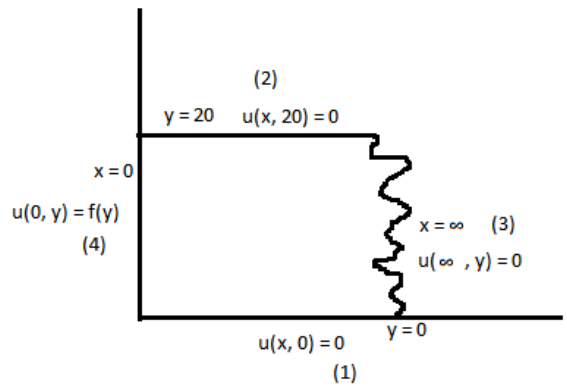
(or)

16.An infinitely long rectangular plate with insulated surface is 20 cm wide .The two long edges and one short edge are kept at $0^{o}c$,while the other short edge x=0 is kept at temperature given by $u=\begin{cases}10y, & 0\le y\le 10\\ 10(20-y), & 10\le y\le 20\end{cases}$ **.Find the steady state temperature in the plate.**

Solution:

The two dimensional equation is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$



The boundary conditions are

1) $u(x,0)=0,\quad 0<x<\infty$

2) $u(x,20)=0,\quad 0<x<\infty$

3) $u(\infty,y)=0,\quad 0<y<20$

4) $u(0,y)=f(y)=\begin{cases}10y, & 0\le y\le 10\\ 10(20-y), & 10\le y\le 20\end{cases}$

Now the possible solutions (trigonometry function in y)as

$$u(x,y)=\left[Ae^{px}+Be^{-px}\right]\left[C\cos py+D\sin py\right]-----(1)$$

Apply 1st Boundary Conditions in [1]

$$u(0,y)=\left[Ae^{px}+Be^{-px}\right]\left[C(1)+D(0)\right]=0$$

$$u(0,y)=\left[Ae^{px}+Be^{-px}\right]C=0$$

But $\left[Ae^{px}+Be^{-px}\right]\neq 0$

$$\therefore\ C=0$$

Sub c=0 in [1]

$$u(x,y)=\left[Ae^{px}+Be^{-px}\right]D\sin py\ -----(2)$$

Apply 2nd Boundary Condition in [2]

$$u(x,20)=\left[Ae^{px}+Be^{-px}\right]D\sin 20p=0$$

But $\left[Ae^{px}+Be^{-px}\right]\neq 0\ \ and\ \ D\neq 0$

$\therefore \ \sin 20p = 0$

$20p = \sin^{-1}(0) = n\pi$

$p = \dfrac{n\pi}{20}$

Sub p value in [2]

$$u(x,y) = \left[Ae^{\left(\frac{n\pi x}{20}\right)} + Be^{-\left(\frac{n\pi x}{20}\right)} \right] D\sin\left(\frac{n\pi y}{20}\right) \text{ -----} (3)$$

Apply 3rd Boundary Condition in [3]

$As \quad x \to \infty \ \Rightarrow \ A = 0$

$\therefore A = 0$

Sub A=0 in [3]

$$u(x,y) = Be^{-\left(\frac{n\pi x}{20}\right)} D\sin\left(\frac{n\pi y}{20}\right)$$

$$u(x,y) = B_n e^{-\left(\frac{n\pi x}{20}\right)} \sin\left(\frac{n\pi y}{20}\right)$$

The most general solution is

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi x}{20}\right)} \sin\left(\frac{n\pi y}{20}\right)} \text{ -------} \rightarrow (4)$$

Apply 4thBoundary Condition in (4)

$$u(0,y) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi y}{20}\right) = f(y) = \begin{cases} 10y, & 0 \le y \le 10 \\ 10(20-y), & 10 \le y \le 20 \end{cases}$$

To find B_n: Expand Fourier sine half range series

$$B_n = \frac{2}{l}\int_0^{\ell} f(x)\sin\frac{n\pi x}{l} \quad \text{dx}$$

$$= \frac{2}{20}\left(\int_0^{10} 10y\sin\left(\frac{n\pi x}{20}\right)dx + \int_{10}^{20} 10(20-y)\sin\left(\frac{n\pi x}{20}\right)dx \right)$$

$$= \frac{10}{10}\left[y\left(-\frac{\cos\frac{n\pi y}{20}}{\frac{n\pi}{20}} \right) - \left(-\frac{\sin\frac{n\pi y}{20}}{\frac{n^2\pi^2}{20^2}} \right) \right]_0^{10} + \left[(20-y)\left(-\frac{\cos\frac{n\pi y}{20}}{\frac{n\pi}{20}} \right) + \left(-\frac{\sin\frac{n\pi y}{20}}{\frac{n^2\pi^2}{20^2}} \right) \right]_{10}^{20}$$

$$= \left[\frac{-10(20)}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{400}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \frac{10(20)}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{400}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) \right]$$

$$= \frac{800}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$$B_n = \frac{800}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

Substitute in equation (4)

$$u(x,y)=\sum_{n=1}^{\infty}\frac{800}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\ e^{-\left(\frac{n\pi x}{20}\right)}\sin\left(\frac{n\pi y}{20}\right)$$

$$u(x,y)=\frac{800}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\ e^{-\left(\frac{n\pi x}{20}\right)}\sin\left(\frac{n\pi y}{20}\right)$$

UNIT IV

17.(a) Fit a parabola to the following data; also estimate y at x=6.

x :	**1**	**2**	**3**	**4**	**5**
y:	**5**	**12**	**26**	**60**	**97**

Solution:

Let, the parabola equation be,

$y = ax^2+bx+c$ ----------(1)

The normal equations are,

$\sum x^2y=a\sum x^4+b\sum x^3+c\sum x^2$ ----------(2)

$\sum xy=a\sum x^3+b\sum x^2+c\sum x$ ----------(3)

$\sum y=a\sum x^2+b\sum x+nc$ ----------(4)

x	**y**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
1	5	1	1	1	5	5
2	12	4	8	16	24	48
3	26	9	27	81	78	234
4	60	16	64	256	240	960
5	97	25	125	625	485	2425
$\sum x$ = 15	**$\sum y$ =**	**$\sum x^2$ = 55**	**$\sum x^3$ =979**	**$\sum x^4$ = 979**	**$\sum xy$ = 832**	**$\sum x^2y$ =3672**

Substitute in eqn (1),(2) and (3)

$3672=979a + 225b + 55c$

$832=225a + 55b + 15c$

$200=55a + 15b + 5c$

Solving this we get

a =0.065, b =24.13, c =-33.1

Sub a,b,c values in $y = ax^2+bx+c$

$y = 0.065x^2 + 24.13x - 33.1$ and when x=6 , y =114.02

(b) A machine puts out 16 imperfect articles in a sample of 500. After the machine is overhauled, it puts out 3 imperfect articles in batch of 100.Has the machine improved?

Solution:

Step 1:

n_1 =500	n_2 =100
$X_1 = 16$	$X_2 = 3$
p_1 =16/500	p_2 =3/100

p_1 =0.032 p_2 =0.03

Step 2:

Null hypothesis, H_0:

$p_1 = p_2$

Alternate hypothesis, H_1 :

$p_1 > p_2$ (Two tailed test)

Step 3: Test Statistic

$$z = \frac{|p_1 - p_2|}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

=((500*0.032)+(100*0.03))/(500+600)

=0.0316

Q=1 – P=0.9684

$$Z = |0.032 - 0.03| / \sqrt{(0.0316 * 0.9684)\left(\frac{1}{500} + \frac{1}{100}\right)}$$

Z = 1.04

Step 4:

At 5% level of significance, Z=1.96

∴calculated value <Tabulated value

i.e H_0 is accepted.

Conclusion:

∴ The proportion of men and women in favour of proposals are same.

(or)

18.(a)Fit a curve of the form y=ab^x to the data.

x :	**2**	**3**	**4**	**5**	**6**
y :	**144**	**172.8**	**207.4**	**248.8**	**298.5**

Solution:

Let, y=ab^x ----------(1)

Taking log on both sides,

log y = log [ab^x]

log y= log a + x log b

Y = A + B x ----------(2)

Where Y=log y ; A=log a ; B=log b

Y=A + B x, which is a straight line equation

The normal equations are,

$\sum xY = A \sum x + B \sum x^2$----------(3)

$\sum Y = n A + B \sum x$ ----------(4)

x	y	Y=log y	x^2	xY
2	144	2.15836	4	**4.316**
3	172.8	2.2375	9	**6.712**
4	207.4	2.3168	16	**9.264**

5	248.8	2.3958	25	11.979
6	298.5	2.4749	36	14.8994
∑x = 20		∑Y = 11.58336	∑ x^2 = 90	∑xY = 45.125

Sub in eqn (3) and (4) we get

47.125 = 20 A + 90 B

11. 58336 =5A + 20 B

A=2.00

B=0.079

Sub in eqn (2)

Y = 1.5069 + 0.1544 x

But a = antilog A

a = 100

b = antilog B

b = 1.199

Sub in eqn (1)

$y = 100\ (1.199)^x$

(b) The following data are got from an investigation.

	No. of cases	**Mean wages**	**S.D of the wages**
Sample I	**400**	**Rs.47.4**	**Rs.3.1**
Sample I	**900**	**Rs.50.3**	**Rs.3.3**

Find out whether the two mean wages differ significantly.

Solution:

Step 1:

$n_1 = 400$ $\quad$ $n_2 = 900$

$\bar{x}_1 = 47.4$ $\quad$ $\bar{x}_2 = 50.3$

$s_1 = 3.1$ $\quad$ $s_2 = 3.3$

Step 2:

Null hypothesis H_0 $\quad \mu_1 = \mu_2$

Alternate hypothesis H_1 $\mu_1 \neq \mu_2$ (two tailed test)

Step 3: Test Statistics

Here σ is not given

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$z = \frac{47.4 - 50.3}{\sqrt{\frac{3.1^2}{400} + \frac{3.3^2}{900}}}$$

$$z = \frac{-2.9}{\sqrt{0.24 + 0.012}} = -5.78$$

Step 4:

The tabulated value of 5% level of significance for two tailed test z_t = 1.96

$$z_c > z_t$$

$\therefore H_0$ is rejected

Conclusion:

$\therefore$ There is no significance difference between two mean wages

UNIT V

19.(a) Two random samples of 11 and 9 items show the sample standard deviation of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the true variances are equal, against the alternative hypothesis that they are not.

Solution:

Given

$$n_1 = 11 \qquad n_2 = 9$$

$$s_1 = 0.8 \quad s_2 = 0.5$$

Here the S.D.'s of the samples s1 and s2 are given. Therefore, the population variance S_1^2 and S_2^2 are obtained by using the relations,

Step 2:

Null Hypothesis H0: The products of both the firms A and B have the same variability.

i.e. $\sigma_1^2 = \sigma_2^2$

Alternative hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$

Step3: the test statistic

$$F = \frac{n_1 s_1^2 / n_1 - 1}{n_2 s_2^2 / n_2 - 1}$$

$$F = \frac{11(0.8)^2 / 11-1}{9(0.5)^2 / 9-1}$$

F = 2.51

i.e. Calculated F = 2.51

Step 4:

Tabulated value of F with (10,8) d.f. at 5% of significance is 3.35

Since calculated F < tabulated F

we accept the null hypothesis H_0.

Conclusion:

The products of both firms A and B have the same variability.

(b) Find if there is any association between extravagance in fathers and extravagance in sons from the following data:

	Extravagant father	**Miserly father**
Extravagant son	**327**	**741**
Miserly son	**545**	**234**

Determine the coefficient of association also.

Solution:

Step 1:

Null hypothesis H_0 : There is an association between extravagance between fathers and sons.

Step 2:

O	327	741	545	234

To find Expected frequency:

	Extravagant father	Miserly father	Total
Extravagant son	327	741	1068
Miserly son	545	234	779
Total	872	975	1847

E(327) = $\frac{872\ X\ 1068}{1847}$= 504.22

E(545) = $\frac{872\ X\ 779}{1847}$ = 367.77

E(741) = $\frac{1068\ X975}{1847}$= 563.77

E(234) = $\frac{975\ X\ 779}{1847}$= 411.22

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
327	504.22	31406.9	62.28
545	367.77	31410.4	85.41
741	563.77	31410.4	55.71
234	411.22	31406.9	76.37

$\Psi^2{}_{cal}$ = 279.77

D.O.F = (m – 1) (n – 1) = 1

$\Psi^2{}_{tab}$ at5% cal = 3.841

$\Psi^2{}_{cal} > \Psi^2{}_{tab}$

H_0 is rejected.

Conclusion:

Hencethere is no association between extravagance between fathers and sons.

(or)

20.Fit a Poisson distribution to the following data and test the goodness of fit.

x:	**0**	**1**	**2**	**3**	**4**	**5**	**6**
f:	**275**	**72**	**30**	**7**	**5**	**2**	**1**

Solution:

Step 1:

Given n = 7

Step 2:

Null hypothesis H_0 : there is no significance difference between O and E

Step 3:

To find expected frequency :

$$f(x) = N\ e^{-\lambda}\frac{\lambda^x}{x!}$$

Here , mean = variance = λ

To find λ:

x	f	fx
0	275	0
1	72	72
2	30	60
3	7	21
4	5	20
5	2	10
6	1	6

Mean $=\frac{\Sigma fx}{\Sigma f}=$ 189/392 = 0.482

λ = 0.482

Here, N $=\Sigma f = 392$

$f(x) = N\ .\ e^{-\lambda}\frac{\lambda^x}{x!}$

$f(0) = 392\ .\ e^{-0.482}\frac{(0.482)^0}{0!} = 242$

$f(1) = 392\ .\ e^{-0.482}\frac{(0.482)^1}{1!} = 117$

$f(2) = 392.\ e^{-0.482}\frac{(0.482)^2}{2!} = 28$

$f(3) = 392\ .\ e^{-0.482}\frac{(0.482)^3}{3!} = 4.52$

$f(4) = 392\ .\ e^{-0.482}\frac{(0.482)^4}{4!} = 0.5$

$f(5) = 392\ .\ e^{-0.482}\frac{(0.482)^5}{5!} = 0.05$

$f(6) = 392\ .\ e^{-0.482}\frac{(0.482)^6}{6!} = 0.004$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
275	242	1089	4.5
72	117	2025	17.3
30	28	4	0.14
7	4.52	6.15	1.36
5	0.5	20.25	40.5
2	.05	3.8025	76.05
1	0.004	0.99	248

$$\Sigma[\frac{(O-E)^2}{E}]=387.85$$

Therefore, $\Psi^2{}_{cal} = 11.07$

Step 4:

D.O.F = n – 2= 7 – 2= 5

Ψ^2_{tab} at5% cal = 11.07

$\Psi^2_{cal} < \Psi^2_{tab}$

Hence H_0 is accepted.

Colclusion:

There is no significance difference between O and E.

MATHEMATICS-IV

FORMULA LIST

UNIT-IV

LARGE SAMPLES

TEST-I

TEST OF SIGNIFICANCE FOR SINGLE PROPORTION

1. To test the significance difference of the sample and population proportion

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

Where p - Sample proportion

P - Population proportion

Q = 1-P

n - Number of samples

2. Confidence limit for population proportion

$P = p \pm 3\sqrt{\frac{pq}{n}}$ When the level of significance (α) is not given

$P = p \pm 2.58\sqrt{\frac{pq}{n}}$ when the level of significance (α)=1% (or) [99% confidence limit]

$P = p \pm 2.33\sqrt{\frac{pq}{n}}$ when the level of significance (α)=2% (or) [98% confidence limit]

$P = p \pm 1.96\sqrt{\frac{pq}{n}}$, When the level of significance (α) = 5% (or) [95% confidence limit]

3. Confidence limit for observed proportion

$$p = P \pm 1.645\sqrt{\frac{PQ}{n}}$$

TEST-II

TEST OF SIGNIFICANCE FOR DIFFERENCE OF PROPORTION

To test the significance difference between,

(i) Two sample proportions

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{Where } P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \; and \; Q = 1 - P$$

$$Z = \frac{P_1 - P_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}}$$ When sample proportions are not given.

(ii) A sample proportion and a combined proportion

$$Z = \frac{p - P}{\sqrt{\frac{n_2 pq}{n_1(n_1 + n_2)}}}$$

(iii) Two population proportion

$$Z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

TEST-III

TEST OF SIGNIFICANCE FOR SINGLE MEAN

(i) To test the significance difference between sample mean and population mean

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Where $\bar{x}$ - Sample mean

μ - Population mean

s- Sample S.D.

σ - Population S.D.

n- Number of Samples.

(ii) Confidence limit for population mean

$\mu = \bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$ When the level of significance (α) = 1% (or) [99% confidence limit]

$\mu = \bar{x} \pm 2.33 \frac{\sigma}{\sqrt{n}}$ When the level of significance (α)=2% (or) [98% confidence limit] $\mu = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$
When the level of significance (α) = 5% (or) [95% confidence limit].

TEST-IV

TEST OF SIGNIFICANCE FOR DIFFERENCE OF MEANS

(i) To test the significance difference between two sample means

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Where $\overline{x_1}$, $\overline{x_2}$ –Sample means

σ_1, σ_2 - Sample S.D.

n_1, n_2 - Size of the samples

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$ Where $\sigma_1 = \sigma_2 = \sigma$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ Where σ_1, σ_2 are not given.

TEST-V

TEST OF SIGNIFICANCE FOR DIFFERENCE OF S.D

To test the significance difference of S.D. of the population

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}}$$

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}}$$ Where σ_1, σ_2 are not given.